Advanced Topics in AI
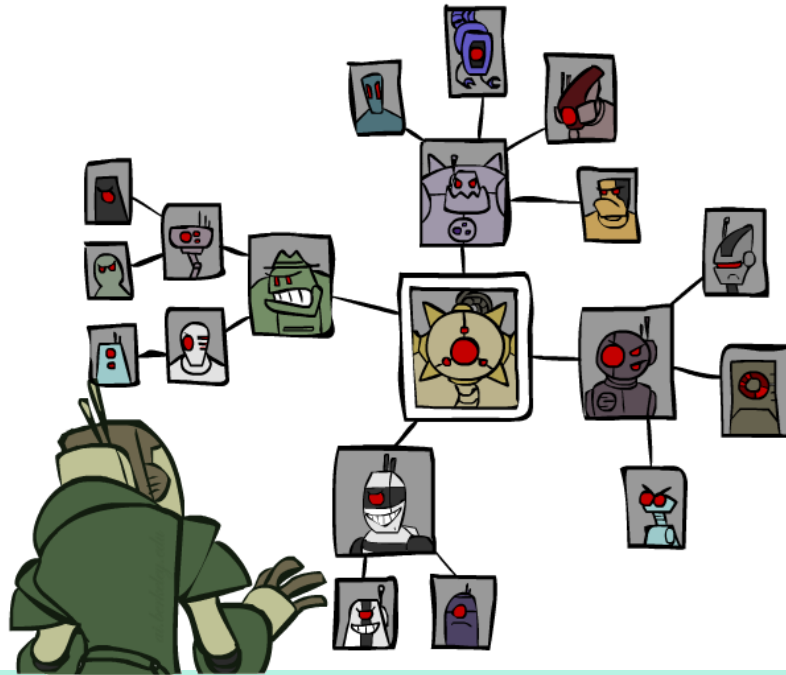## Tree-Structured CSPs

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Problem Structure

- Extreme case: independent subproblems
  - Example: Tasmania and mainland do not interact

- Independent subproblems are identifiable as connected components of constraint graph

- Suppose a graph of n variables can be broken into subproblems of only c variables:
  - Worst-case solution cost is $O((n/c)(d^c))$, linear in n
  - E.g., n = 80, d = 2, c =20
  - 2^{80} = 4 billion years at 10 million nodes/sec
 - $(4)(2^{20})$ = 0.4 seconds at 10 million nodes/sec
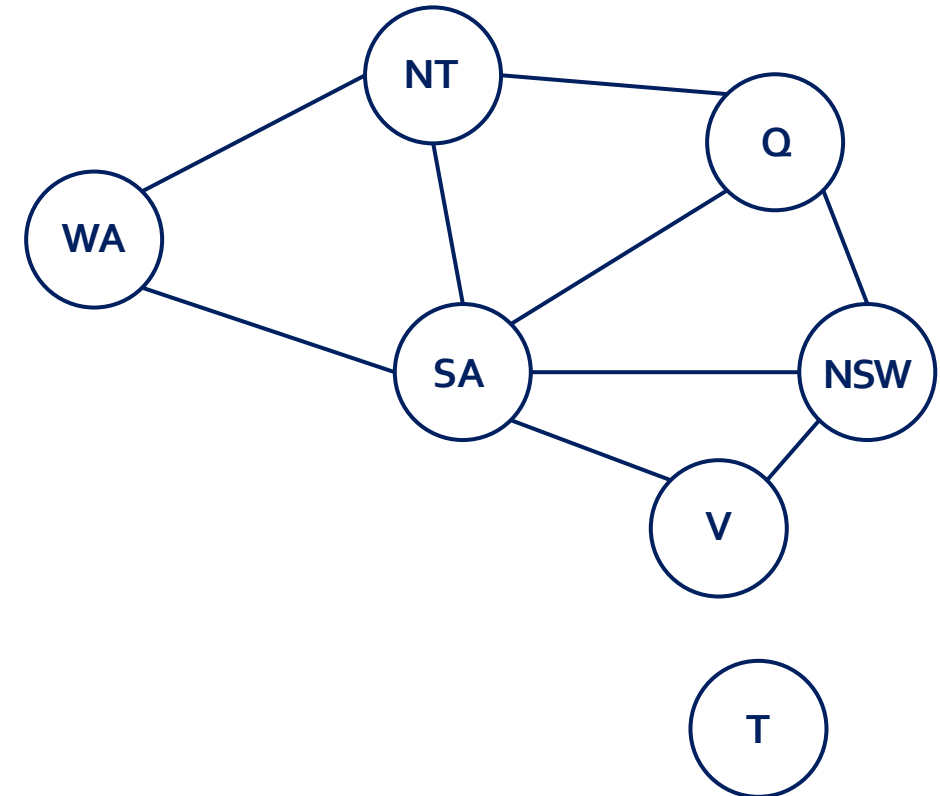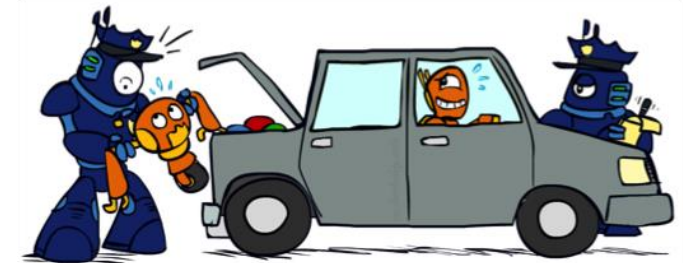
Tree-Structured CSPs
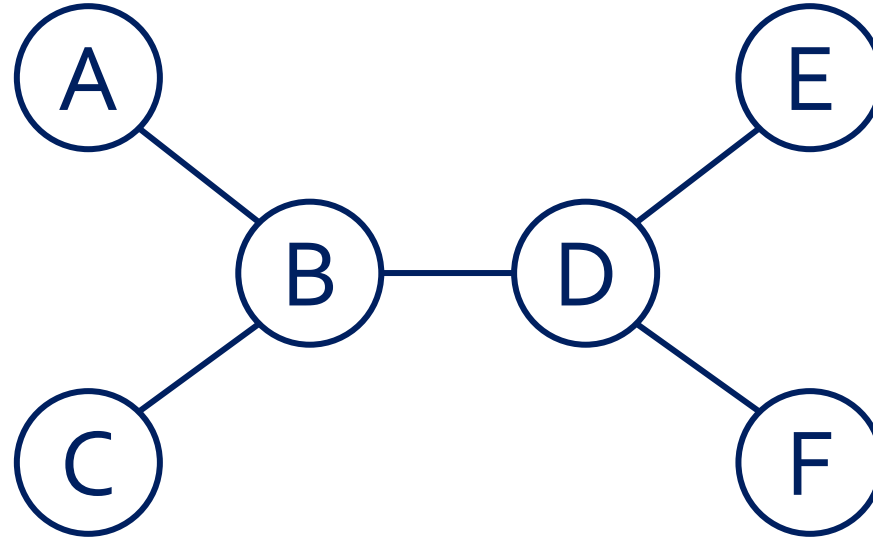
- Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\,d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$

- This property also applies to probabilistic reasoning (later): an example of the relation between syntactic restrictions and the complexity of reasoning

Tree-Structured CSPs

- Algorithm for tree-structured CSPs:
  - Order: Choose a root variable, order variables so that parents precede children
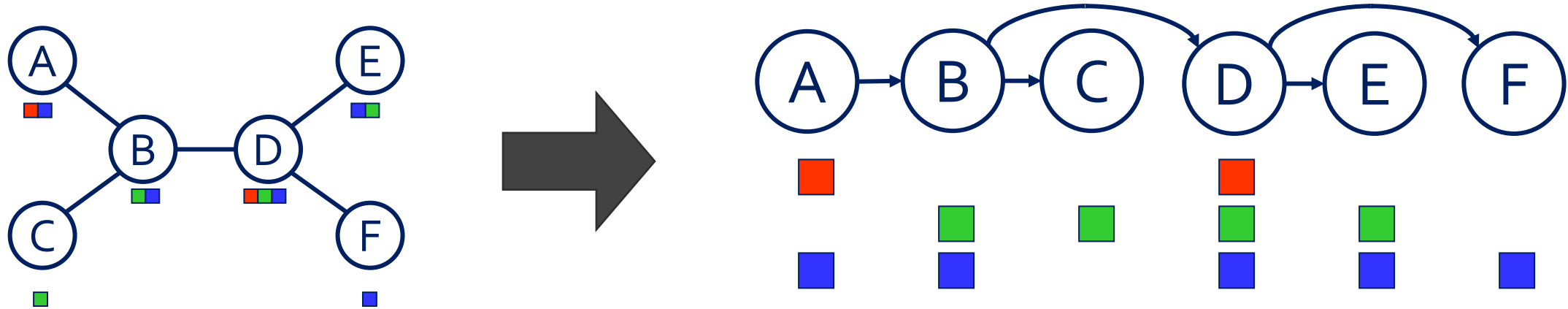

- Remove backward: For i = n : 2, apply REMOVEINCONSISTENT(Parent(X_i),X_i)
- Assign forward: For i = 1 : n, assign X_i consistently with Parent(X_i)

- Runtime: O(n d^2) (why?)

Tree-Structured CSPs

- **Claim 1**: After backward pass, all root-to-leaf arcs are consistent
- **Proof**: Each X→Y was made consistent at one point and Y's domain could not have been reduced thereafter (because Y's children were processed before Y)
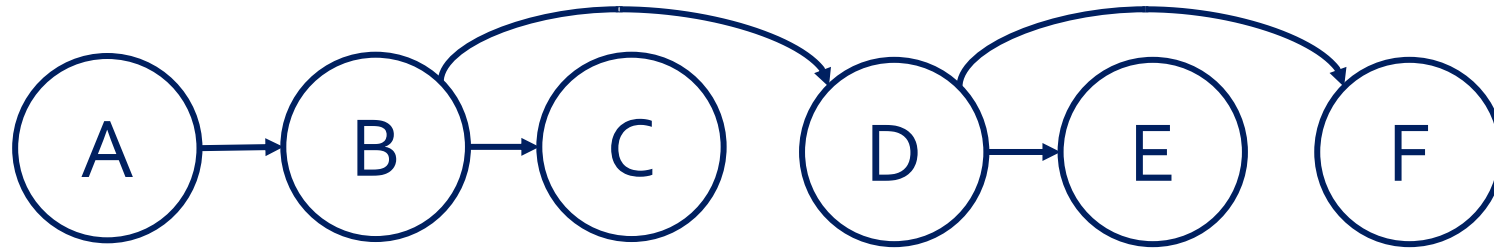
- **Claim 2**: If root-to-leaf arcs are consistent, forward assignment will not backtrack
- **Proof**: Induction on position

- Why doesn't this algorithm work with cycles in the constraint graph?

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Next: Improving structure

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