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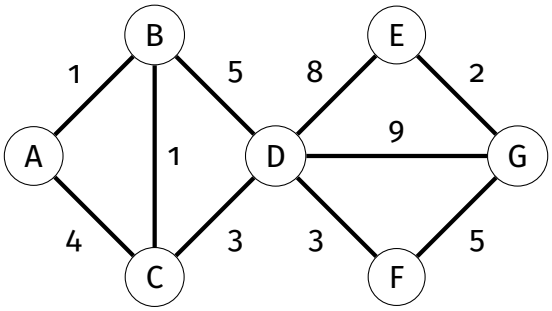
October 4th 2023

Advanced Topics in AI
Exercise 1 - Search

Question 1: Search Algorithms



Node	h_1	h_2
A	9.5	10
B	9	12
C	8	10
D	7	8
E	1.5	1
F	4	4.5
G	0	0

Consider the state space graph shown above. A is the start state and G is the goal state. The costs for each edge are shown on the graph. Each edge can be traversed in both directions. Note that the heuristic h_1 is consistent but the heuristic h_2 is not consistent.

Question 1.1: Properties

Which properties do depth-first search, breadth-first search, uniform cost search, greedy search and A* search have? Describe the approach of the methods and name the advantages and disadvantages.

Question 1.2: Possible paths returned

For each of the following graph search strategies *(do not answer for tree search)*, mark which, if any, of the listed paths it could return. Note that for some search strategies the specific path returned might depend on tie-breaking behavior. In any such cases, make sure to mark all paths that could be returned under some tie-breaking scheme.

Search algorithm	A-B-D-G	A-C-D-G	A-B-C-D-F-G
Depth first search			
Breath first search			
Uniform cost search			
A* search with heuristic h_1			
A* search with heuristic h_2			

Question 1.3: Heuristic function properties

Suppose you are completing the new heuristic function h_3 shown below. All the values except $h_3(B)$ are fixed and satisfy the consistency conditions.

Node	A	B	C	D	E	F	G
h_3	10	?	9	7	1.5	4.5	0

For each of the following conditions, write the set of values that are possible for $h_3(B)$. For example, to denote all non-negative numbers, write $[0, \infty)$, to denote the empty set, write $\varnothing$, and so on.

1. What values of $h_3(B)$ make h_3 admissible?
2. What values of $h_3(B)$ make h_3 consistent?
3. What values of $h_3(B)$ will cause A* graph search to expand node A, then node C, then node B, then node D in order.

Question 2: n-Pac-Men search

Consider the problem of controlling n Pac-Men simultaneously. Several Pac-Men can be in the same square at the same time, and at each time step, each Pac-Man moves by at most one unit vertically or horizontally (in other words, a Pac-Man can stop, and also several Pac-Men can move simultaneously). The goal of the game is to have all the Pac-Men be at the same square in the minimum number of time steps. In this question, use the following notation: let M denote the number of squares in the maze that are not walls (i.e. the number of squares where Pac-Men can go); n the number of Pac-Men; and $p_i = (x_i, y_i) : i = 1...n$, the position of Pac-Man i. Assume that the maze is connected.

1. What is the state space of this problem?
2. What is the size of the state space (not a bound, the exact size)?
3. Give the tightest upper bound on the branching factor of this problem.
4. Bound the number of nodes expanded by uniform cost tree search on this problem, as a function of n and M. Justify your answer.
5. Which of the following heuristics are admissible? Which one(s), if any, are consistent? Briefly justify all your answers.
   a) The number of (ordered) pairs (i, j) of Pac-Men with different coordinates:

$$h_1(p_1, ..., p_n) = \Sigma^n{}_{i=1}\Sigma^n{}_{j=i+1}\mathbf{1}[p_i \neq p_j] \quad where \quad \mathbf{1}[p_i \neq p_j] = \begin{cases} 1 & if \ p_i \neq p_j \\ 0 & otherwise \end{cases}$$

   b) $h_2((x_1, y_1), ..., (x_n, y_n)) = \frac{1}{2} max \left\{ max_{i,j} \ |x_i - x_j|, max_{i,j} \ |y_i - y_j| \right\}$

Question 3: Travelling through Romania

Perform a search to find a way through Romania. Document the process in a table using the following table schema:

Step	Fringe	Explored	Children

The columns Fringe and Explored should contain the sets at the beginning of each step, the column Children should contain the set that is created during the step.
Use the street map shown in Figure 1 and the heuristic shown in Figure 2.

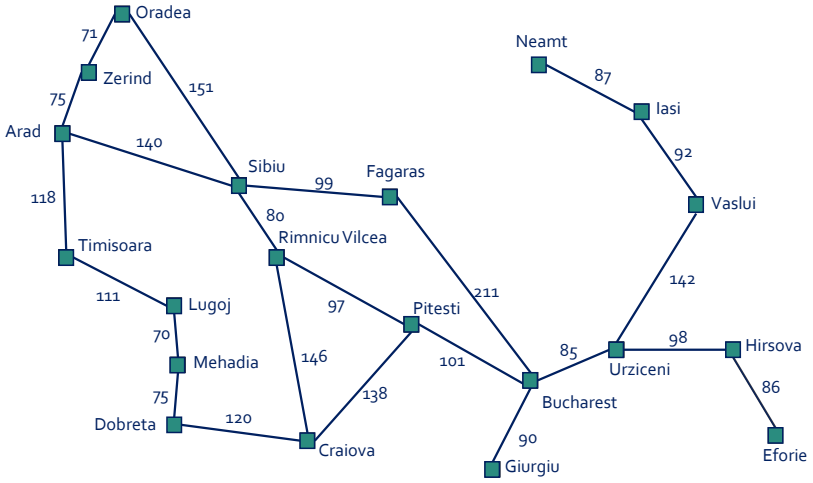


Figure 1: Simplified street map of Romania

Arad	366	Mehdia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Dobreta	242	Pitesti	100
Eforie	161	Riminicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

Figure 2: Values for the ideal distance to Bucharest

1. Perform tree-search Depth-First Search from Arad to Bucharest.
2. Perform tree-search Breadth-First Search from Arad to Bucharest.

3. Perform tree-search Uniform Cost Search from Sibiu to Bucharest.
4. Perform tree-search A* Search from Arad to Bucharest