Advanced Topics in AI
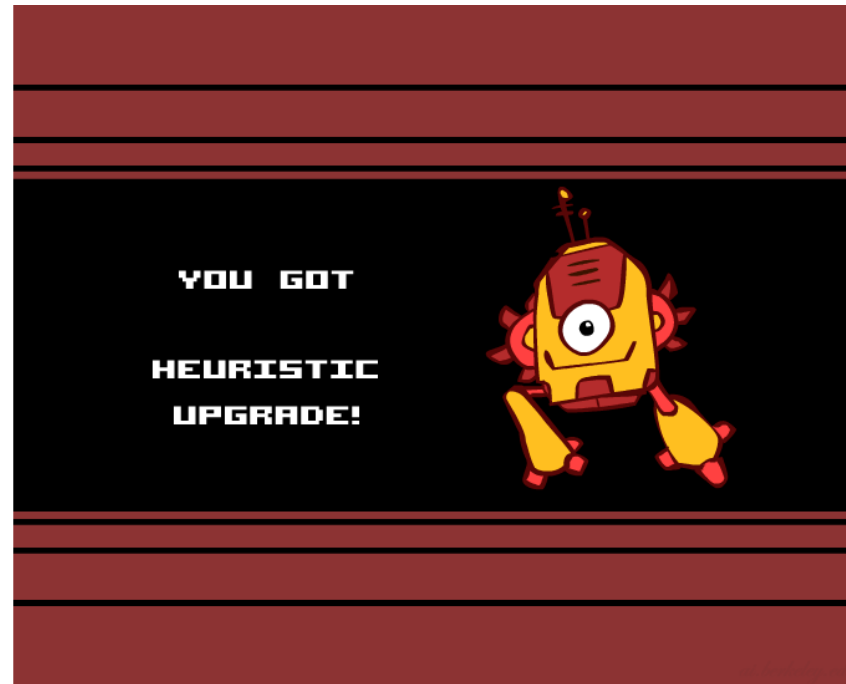## Creating Heuristics

Instructor: Prof. Dr. techn. Wolfgang Nejdl

Leibniz University Hannover
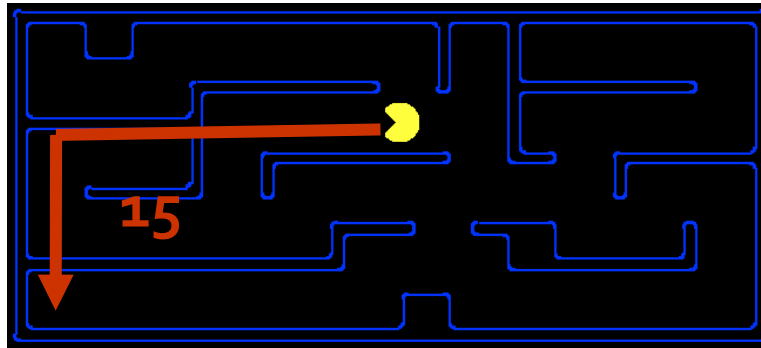
Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available

- Inadmissible heuristics are often useful too

Creating Admissible Heuristics

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
 - What's wrong with it?
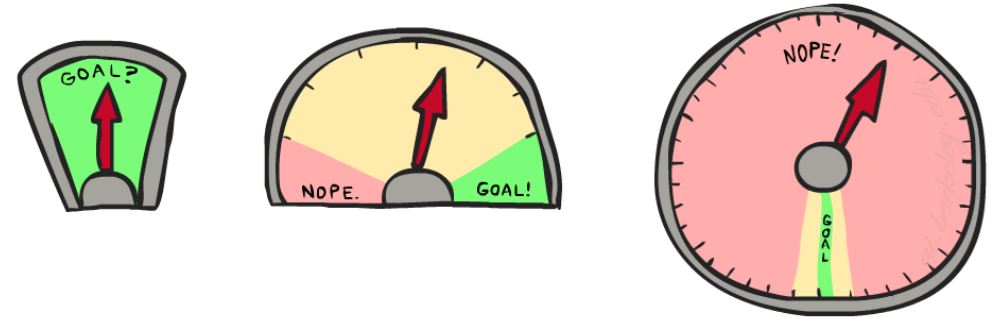
- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Semi-Lattice of Heuristics
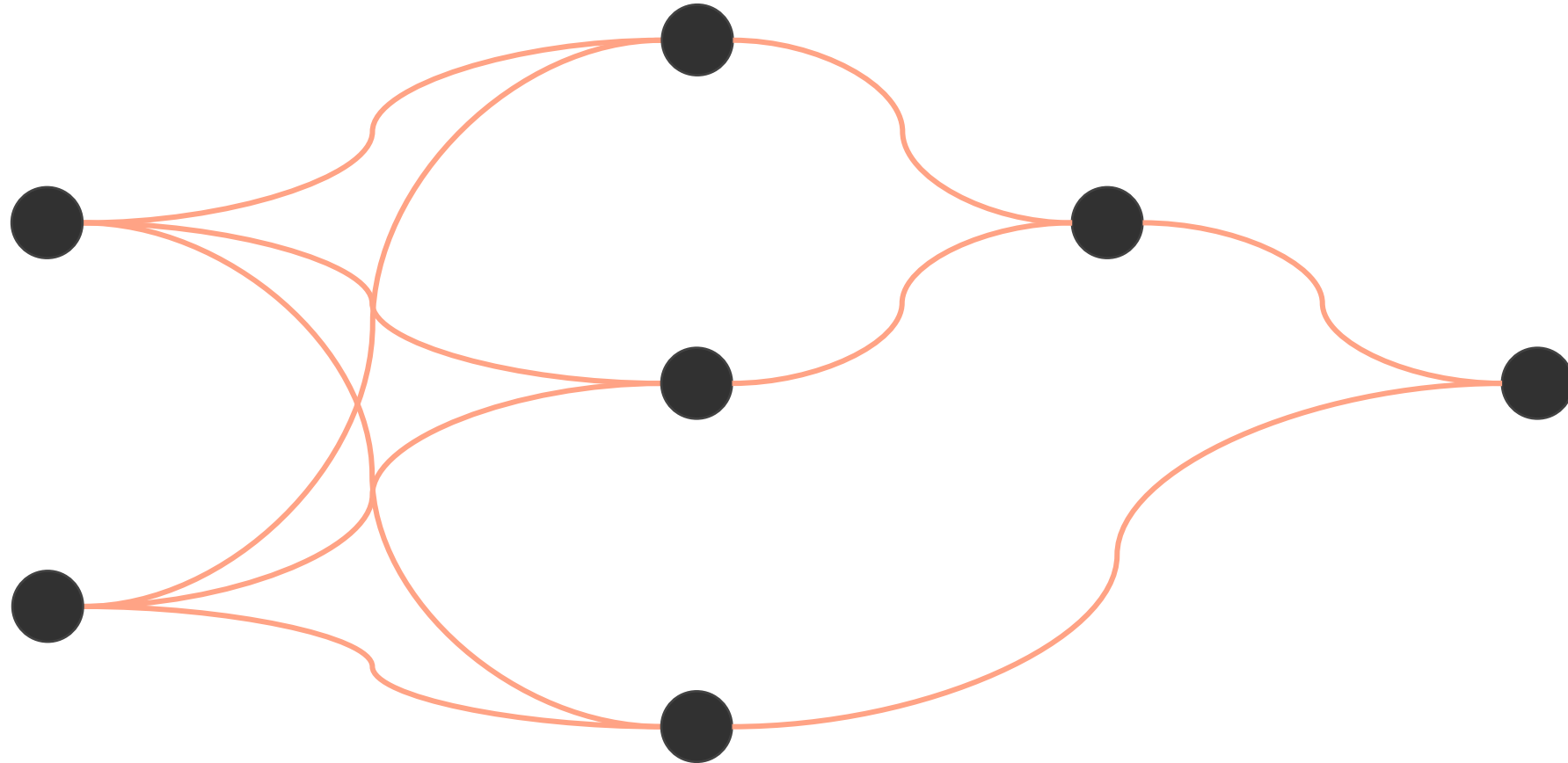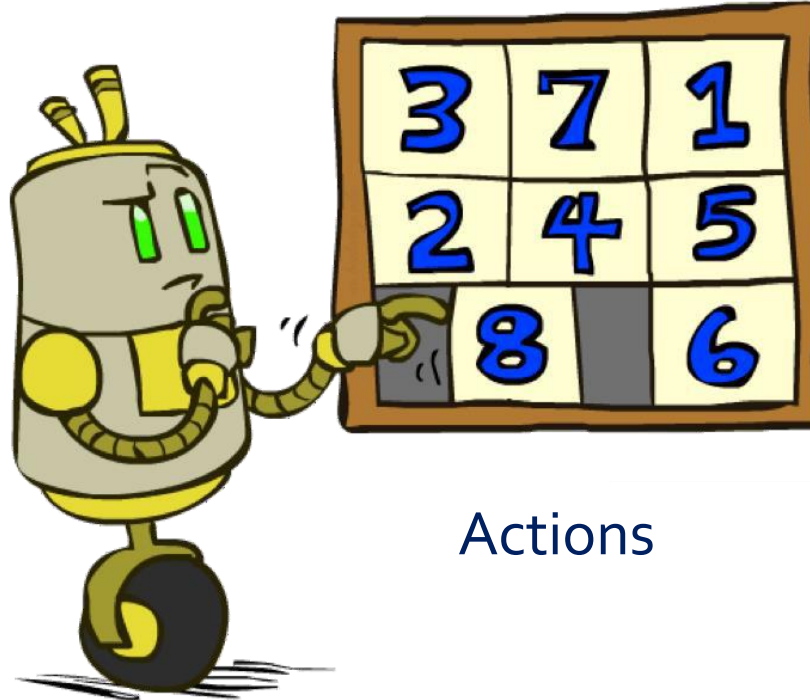
Trivial Heuristics, Dominance

- Dominance: $h_a \geq h_c$ if
$$\forall n: h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible
  $$h(n) = \max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic

$exact$

$|$

$max(h_a, h_b)$

$h_a \qquad h_b$

h_c

$zero$
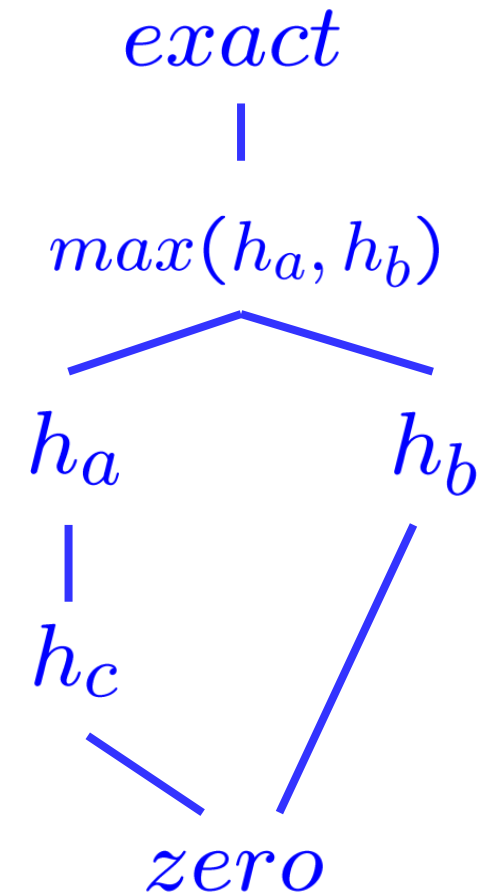
Example: 8 Puzzle

Start State

Actions

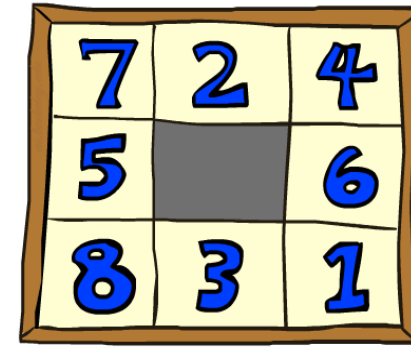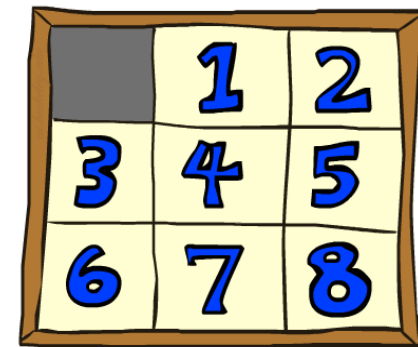**Goal State**

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
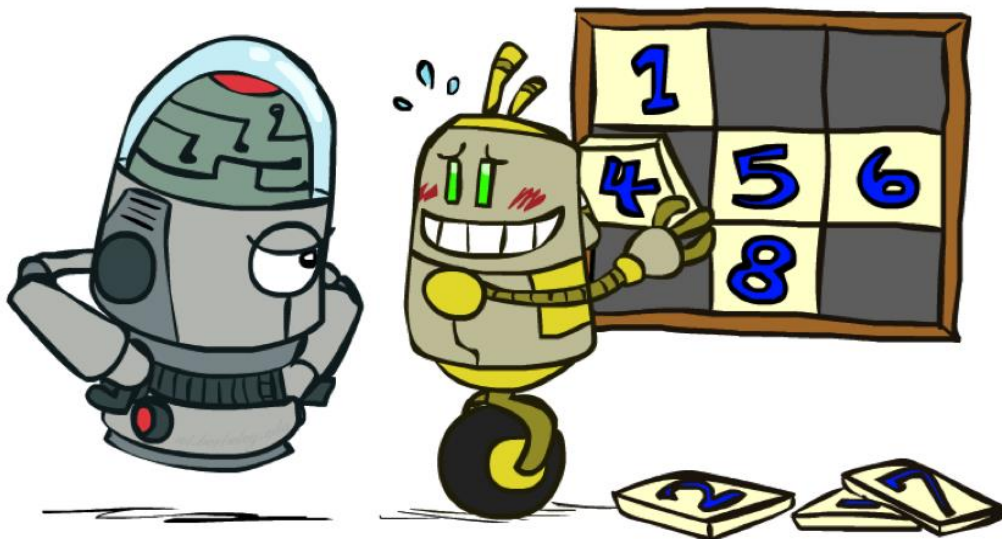- What should the costs be?

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) =   8
- This is a relaxed-problem heuristic

Start State   Goal State

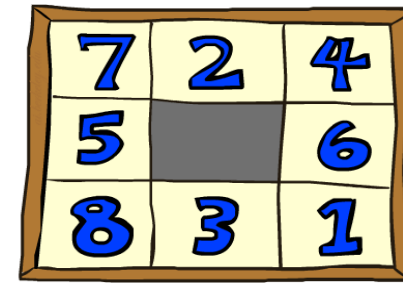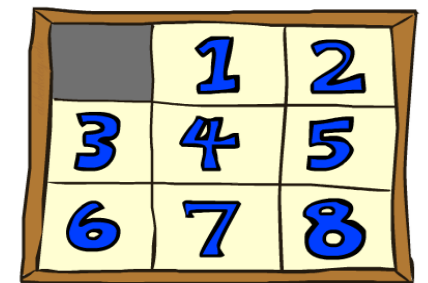| Average nodes expanded when the optimal path has…		
…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6 x 10^6
TILES	13	39	227

Statistics from Andrew Moore

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?

- Total Manhattan distance

- Why is it admissible?
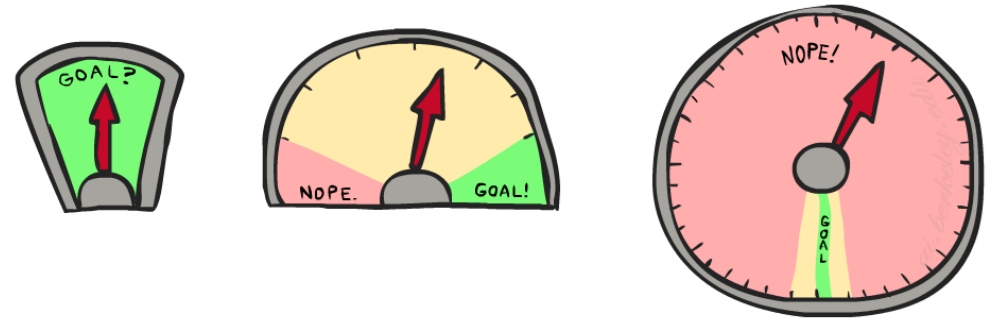
- h(start) =   3 + 1 + 2 + … = 18

Start State   Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MANHATTAN	12	25	73

8 Puzzle III

- How about using the actual cost as a heuristic?
  - Would it be admissible?
 - Would we save on nodes expanded?
  - What's wrong with it?

- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Optimality

- Tree search:
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)

- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)

- Consistency implies admissibility

- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems
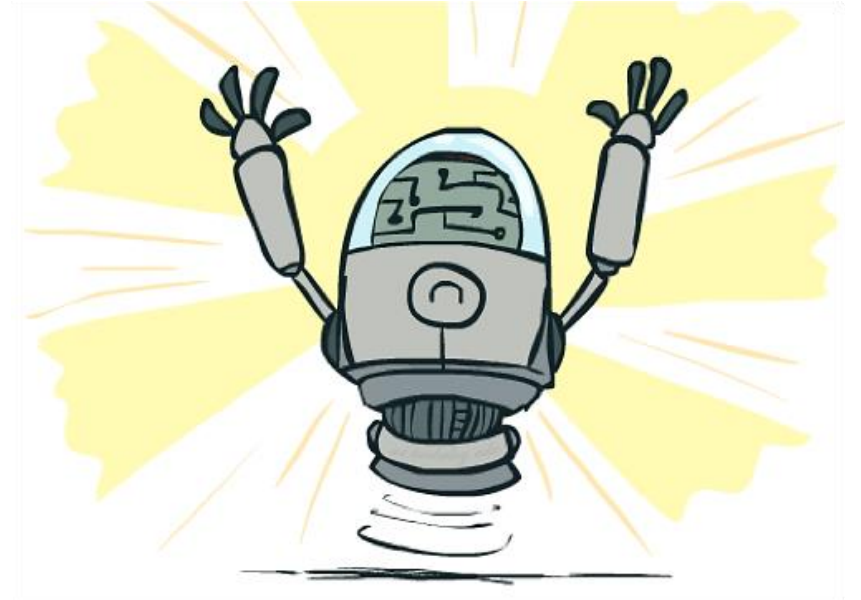
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Next: Summary and Conclusion

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Leibniz University Hannover