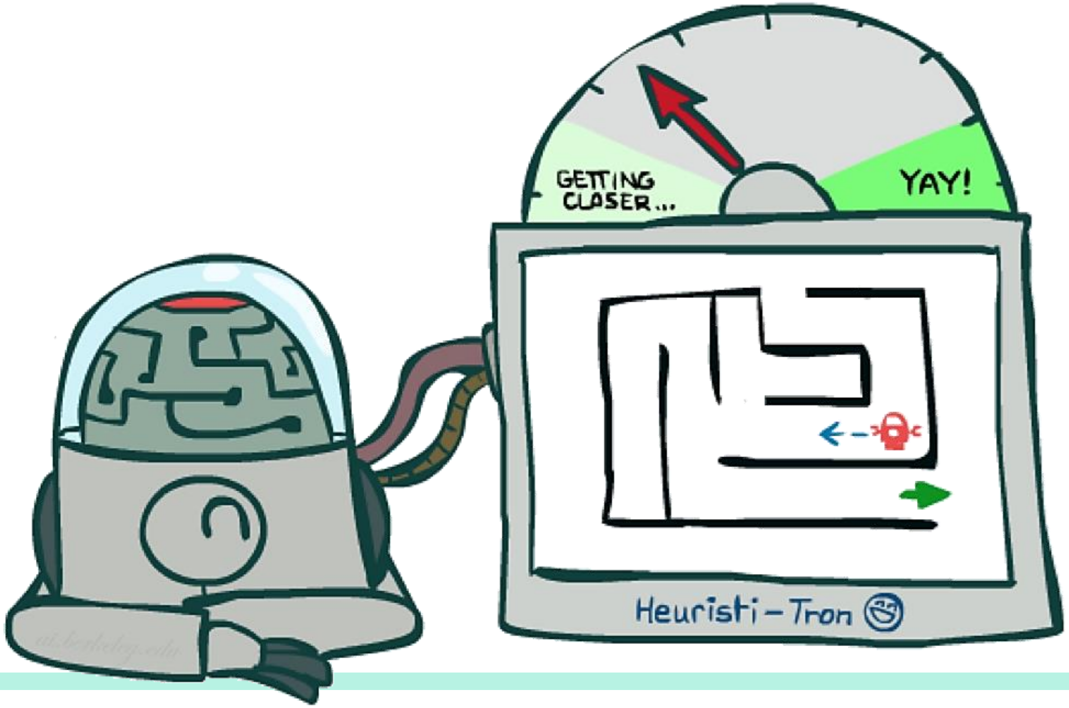
Advanced Topics in AI
Heuristics



Instructor: Prof. Dr. techn. Wolfgang Nejdl

Leibniz University Hannover

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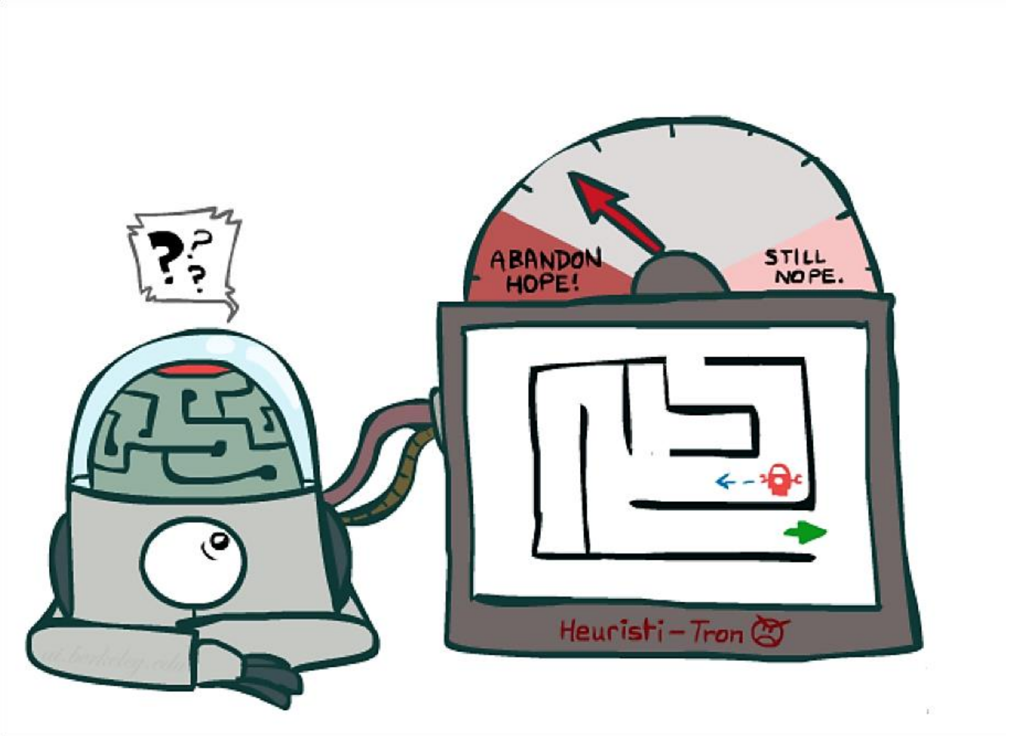
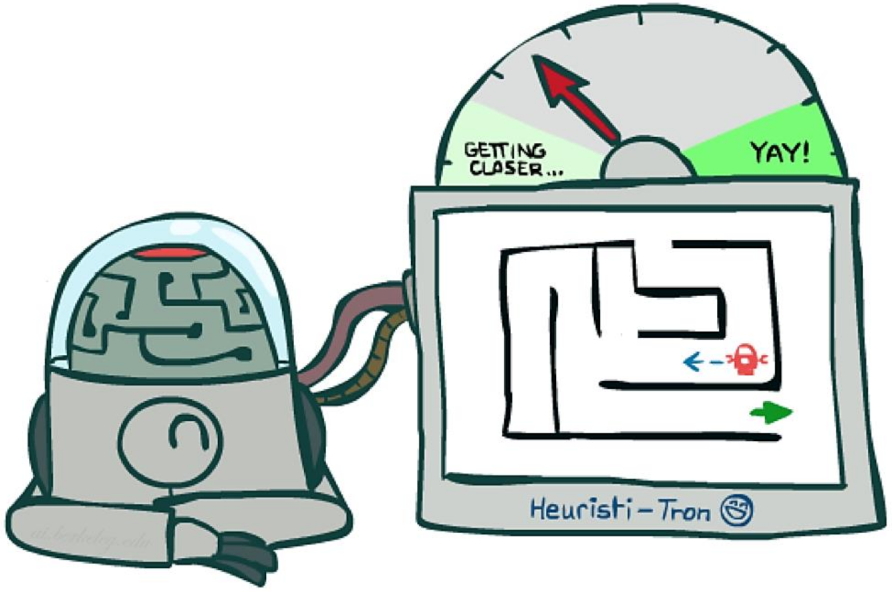
Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

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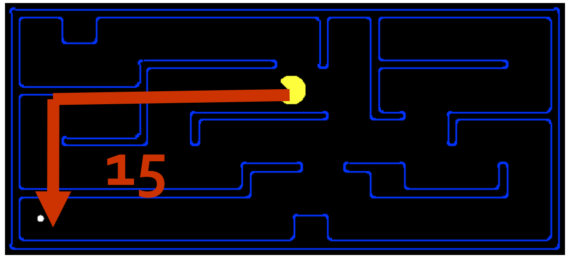
Admissible Heuristics

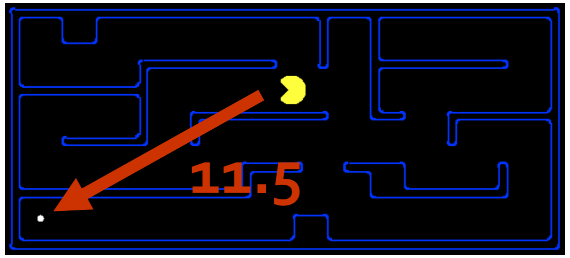
- A heuristic *h* is *admissible* (optimistic) if:

$$0 \le h(n) \le h^*(n)$$

  where $h^*(n)$ is the true cost to a nearest goal

- Examples:





0.0

- Coming up with admissible heuristics is most of what's involved in using A* in practice.

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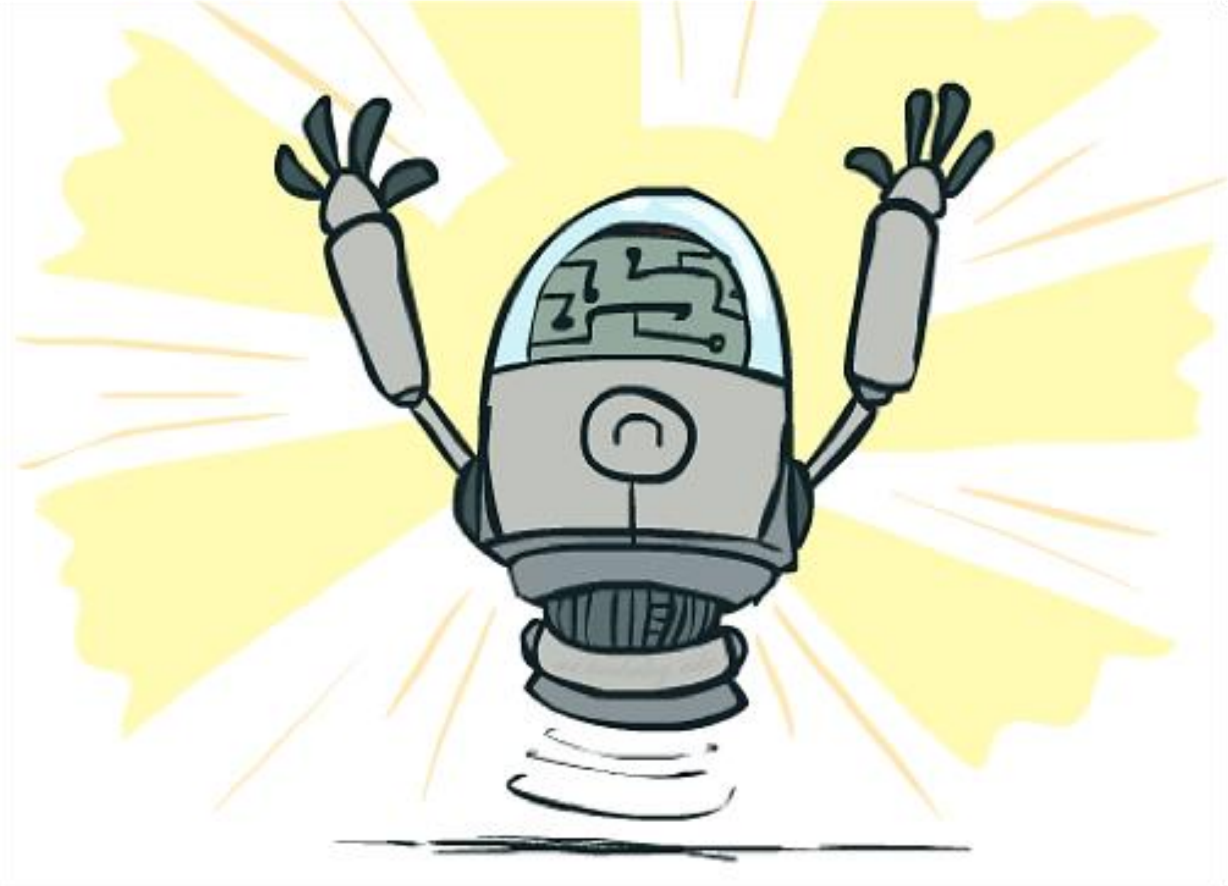
Optimality of A* Tree Search



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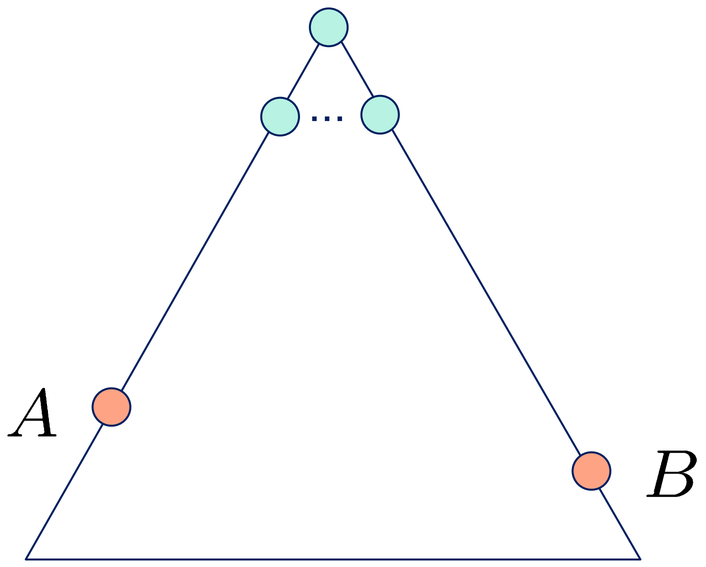
Optimality of A* Tree Search

- Assume:
  - A is an optimal goal node
  - B is a suboptimal goal node
  - h is admissible



- Claim:
  - A will exit the fringe before B

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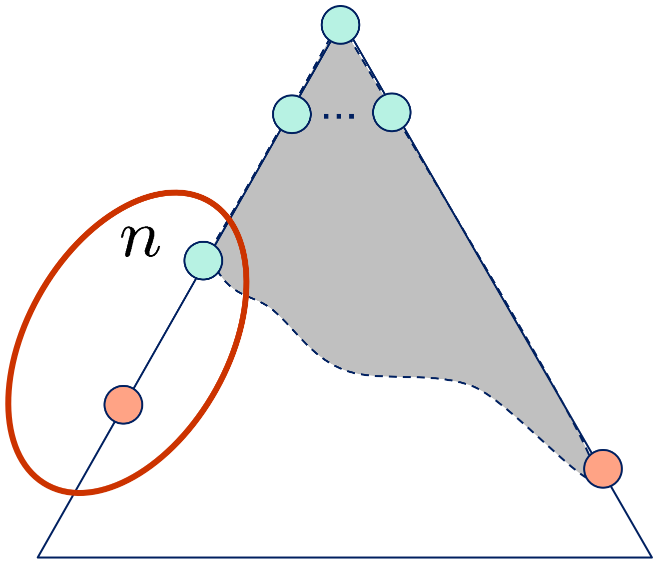
Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)



$f(n) = g(n) + h(n)$	Definition of f-cost
$f(n) \leq g(A)$	Admissibility of h
$g(A) = f(A)$	$h = 0$ at a goal

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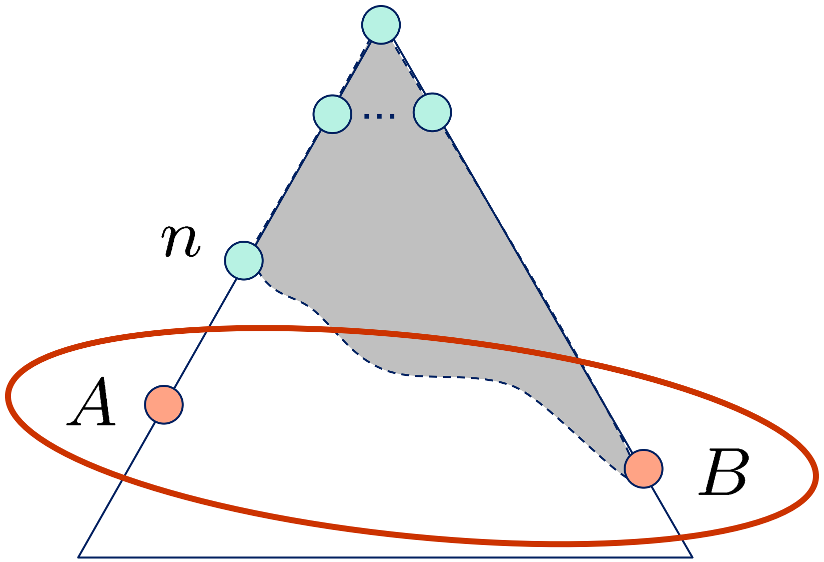
Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)



$g(A) < g(B)$ B is suboptimal

$f(A) < f(B)$ $h = 0$ at a goal

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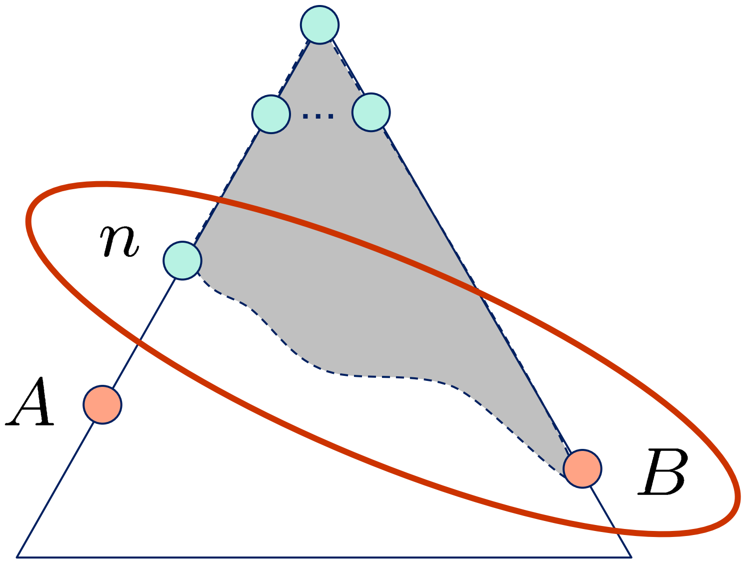
Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)
  3. *n* expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal



$$f(n) \leq f(A) < f(B)$$

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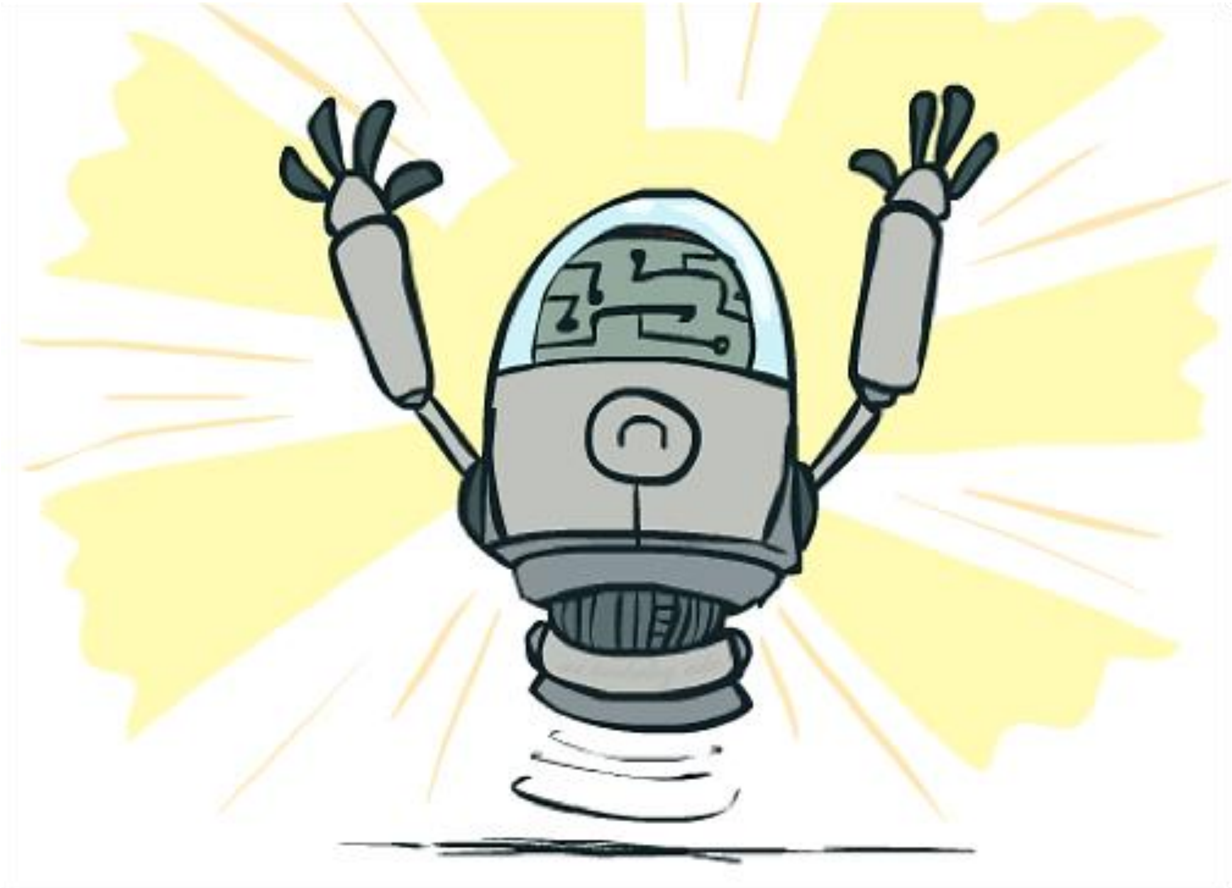
Optimality of A* Graph Search



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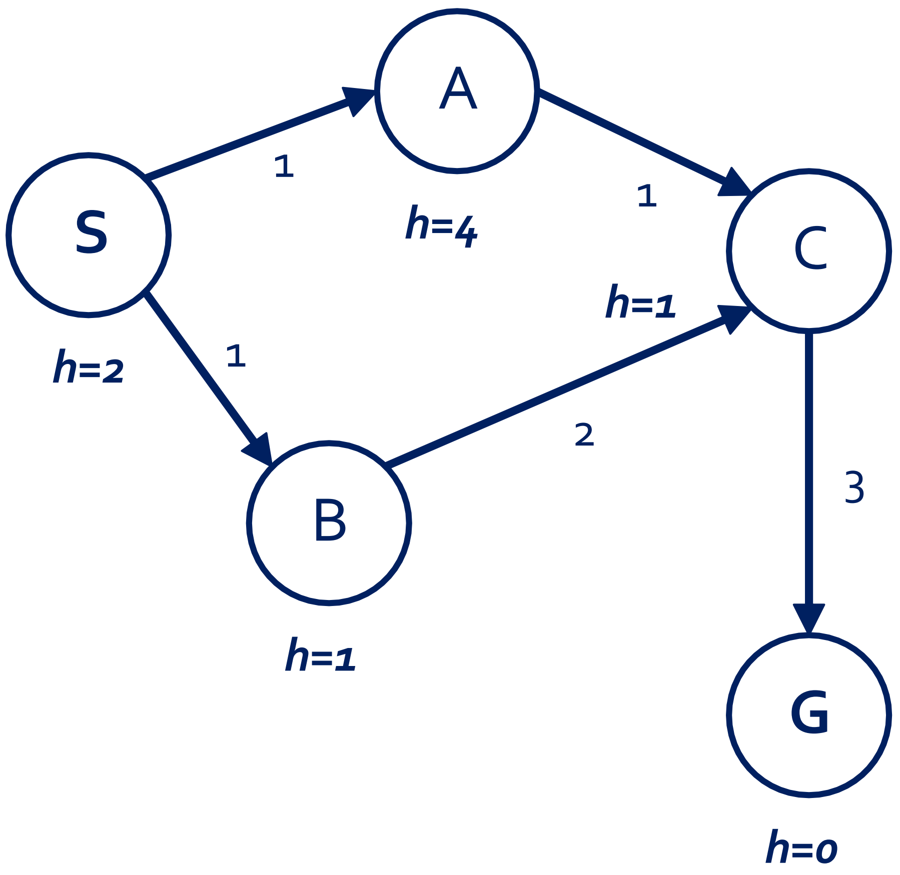
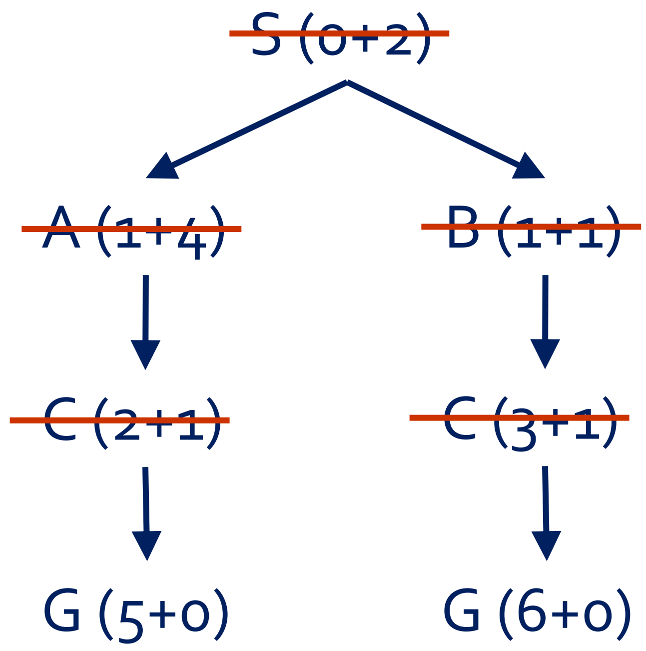
A* Graph Search Gone Wrong?

State space graph



Search tree



Closed Set: S B C A

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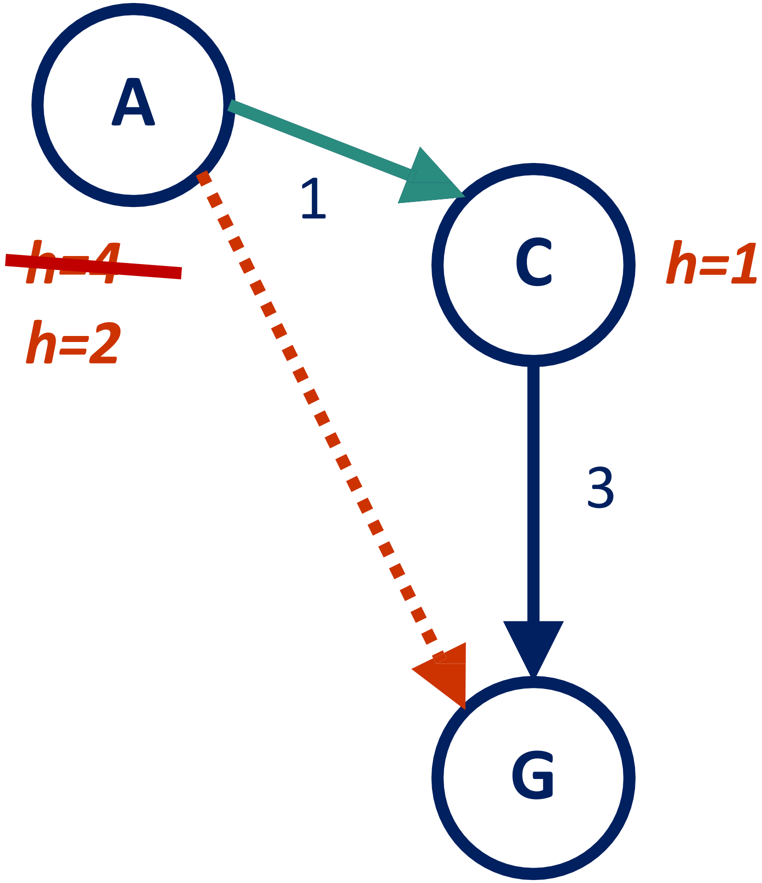
Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G

  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)

- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)

  - A* graph search is optimal

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Optimality of A* Search

- With an admissible heuristic, Tree A* is optimal.
- With a consistent heuristic, Graph A* is optimal.
- With h=0, the same proof shows that UCS is optimal.

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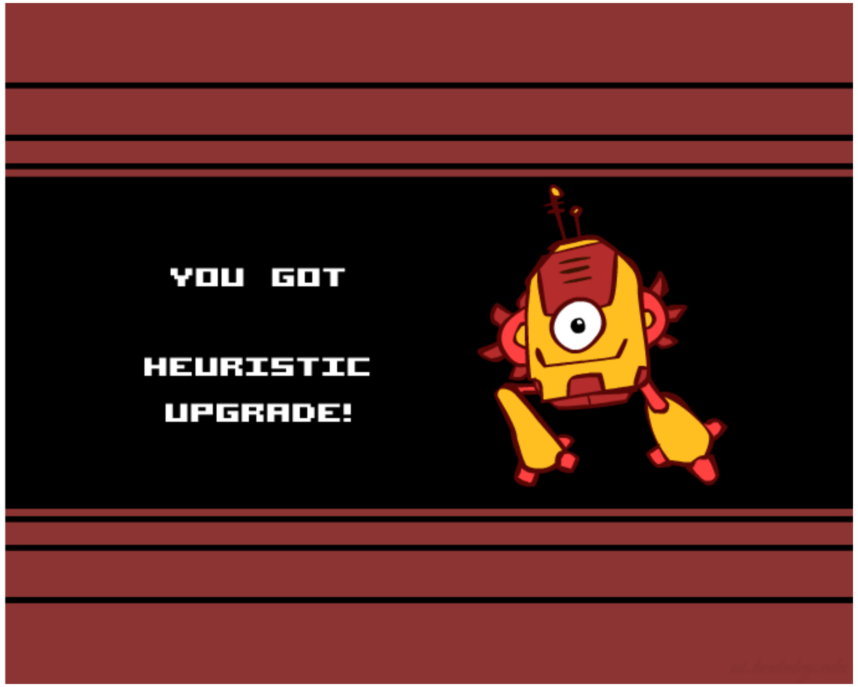
Advanced Topics in AI

Next: Creating Heuristics



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[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]

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