Proof Slide 3

Let

$$Var(Y) = \sum_{j=1}^{p} \beta_j Cov(X_j, Y) + Var(\varepsilon).$$

If $\beta_j = \dfrac{Cov(X_j, Y)}{Var(X_j)}$, then

$$Var(Y) = \sum_{j=1}^{p} \frac{[Cov(X_j, Y)]^2}{Var(X_j)} + Var(\varepsilon).$$

As $\rho^2_{X_j,Y} = \frac{[Cov(X_j,Y)]^2}{Var(X_j)Var(Y)}$, it results that

$$Var(Y) = \sum_{j=1}^{p} Var(Y)\rho^2_{X_j,Y} + Var(\varepsilon).$$

Moreover, $R^2 = \frac{Var(\hat{Y})}{Var(Y)} = \frac{\sum_{j=1}^{p} \rho^2_{X_j,Y} Var(Y)}{Var(Y)} = \sum_{j=1}^{p} \rho^2_{X_j,Y} = \sum_{j=1}^{p} Var(\hat{Y}_j)$.