Image, filtering, convolution

Computer Vision

Winter Semester 20/21

Goethe University

Acknowledgement:
Some images are from various sources: UCF, Stanford cs231n, etc.

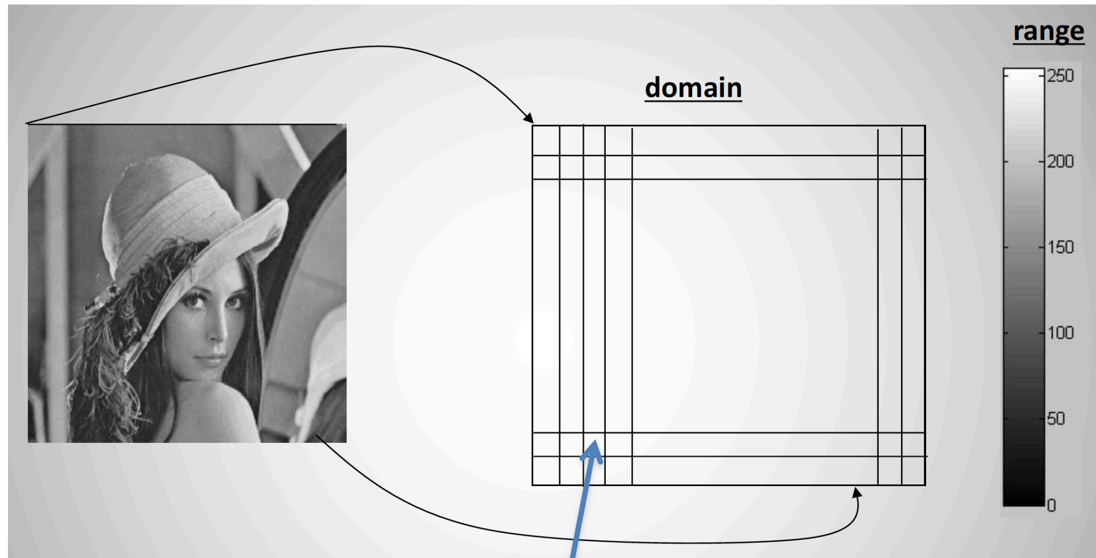
Image is an array of numbers

-Grayscale image

-2D array of numbers (pixels) / matrix

-Number indicates the intensity: [0,255] for 8-bit representation

-Image resolution / number of pixel in an image: 100x100, 1920x1080, etc.



A pixel

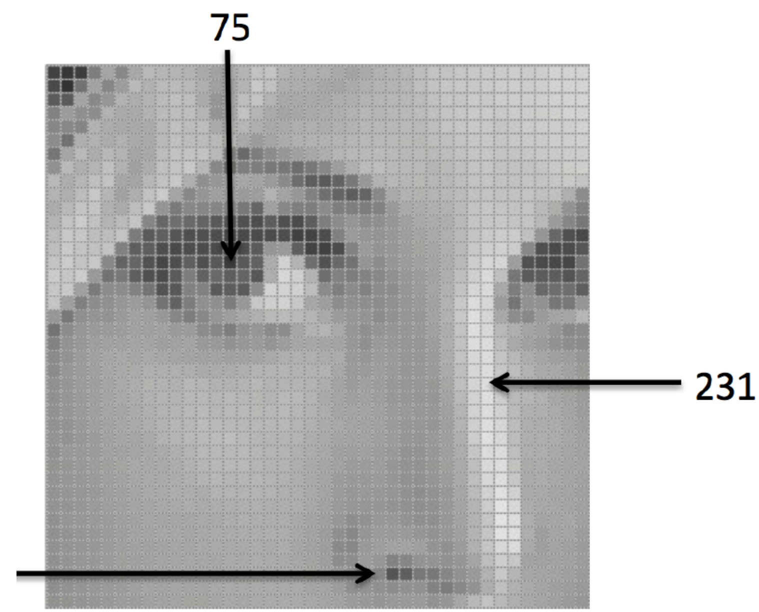
0: black, 255: white

Image is a discrete array of numbers

-Samples from continuous object

-Quantized to have a finite number of possible values: [0,1,2 to 255]

- Sometimes normalized from 0 to 1



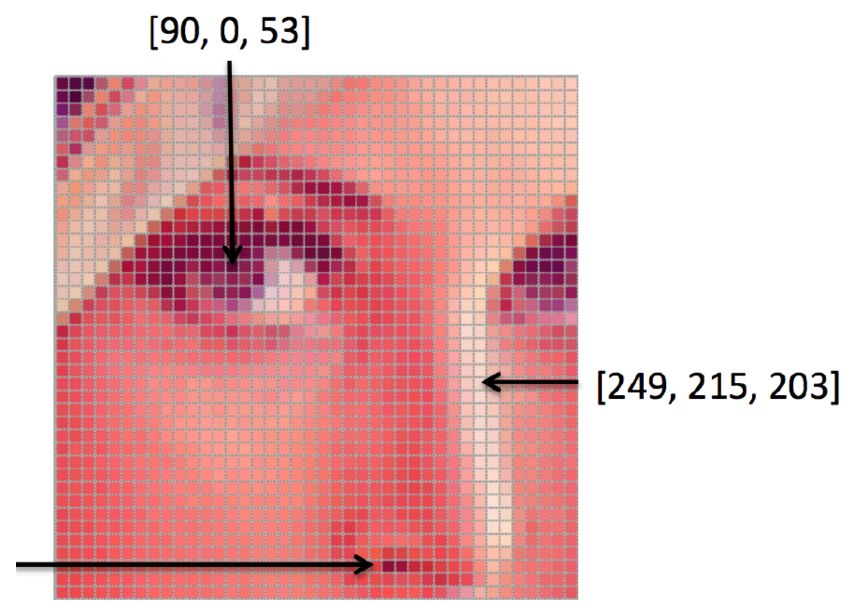
Color image

-Each pixel has three numbers to represent the red, green, blue color intensity

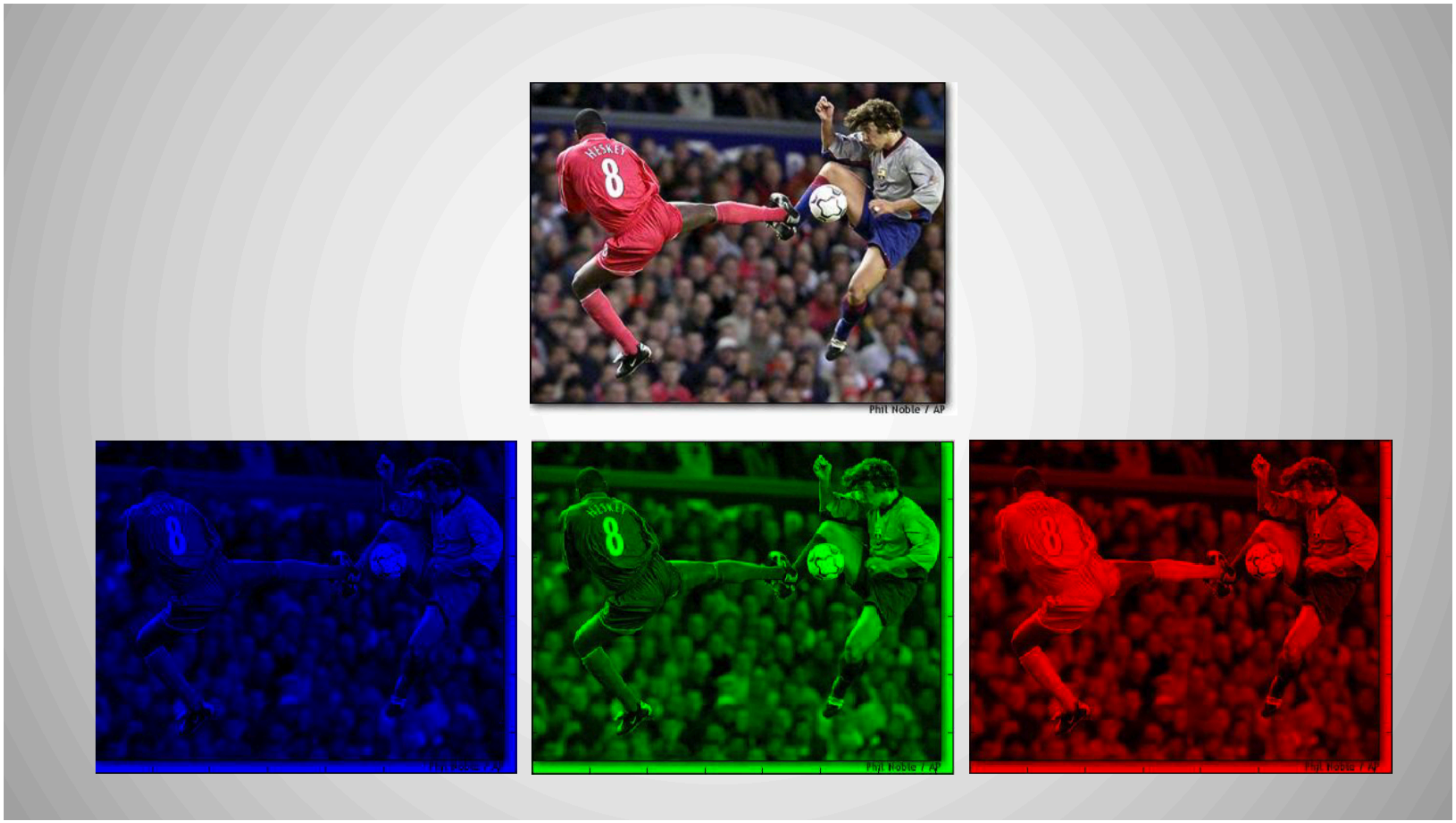
-RGB (Red, Green, Blue)

-Other color format:
  * HSV (Hue, Saturation, Value)
  * YUV (luma component (Y), blue projection (U), red projection (V)

-Image can be stored in different format: JPEG, PNG, TIF, etc.

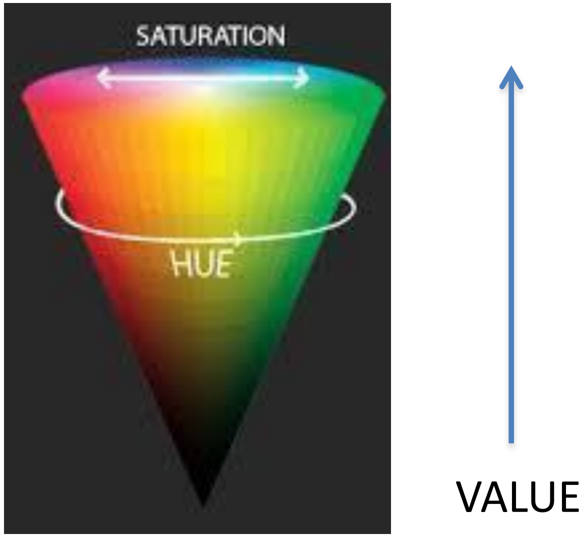


Color image with R, G, B channels



HSV color space

- **Hue** represents color in angle of HSV cone
  - 0-60: red
  - 60-120: yellow
  - 120-180: green
  - 180-240: cyan
  - 240-300: blue
  - 300-360: magenta
- **Saturation** represents the amount of ‘grey’ in the color: ‘0’ is grey, ‘1’ is pure primary color
  - Small saturation means faded color
  - It is the radius of the HSV cone
- **Value** represents intensity
  - It is the height of the HSV cone



HSV represents color in a similar way as humans perceive color, is easy to work with in some applications

Video

- A video is a sequence of images (frames)
- 30 frame per second (fps)
- Spatial dimension: x,y
- Temporal dimension: t

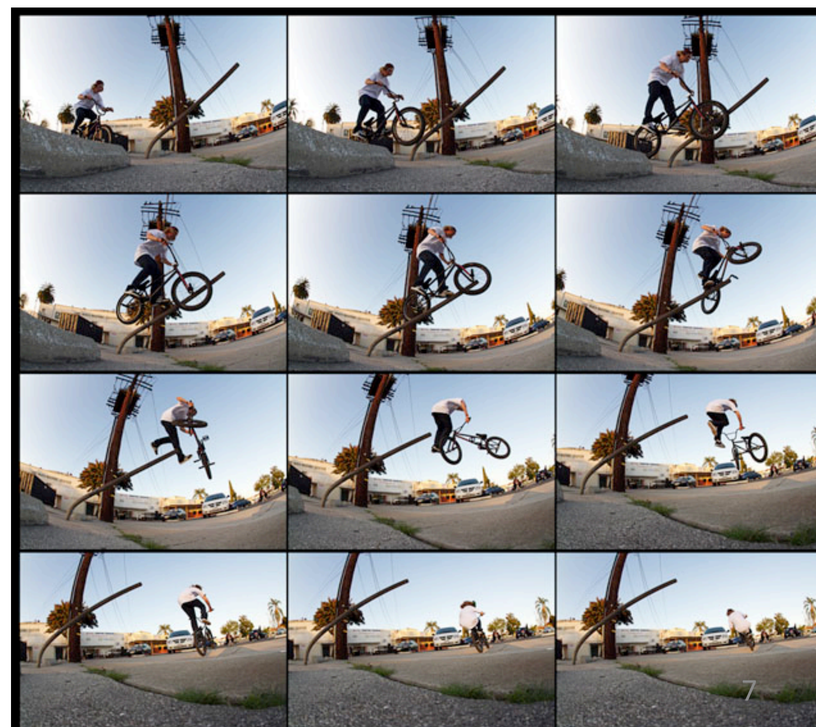


Image Filtering

- An image processing operation
- Remove some unwanted components: noise
- Extract useful information
- Linear filtering: The output is a linear combination of pixel values in some neighborhood

Filtering is an essential operation in deep neural networks

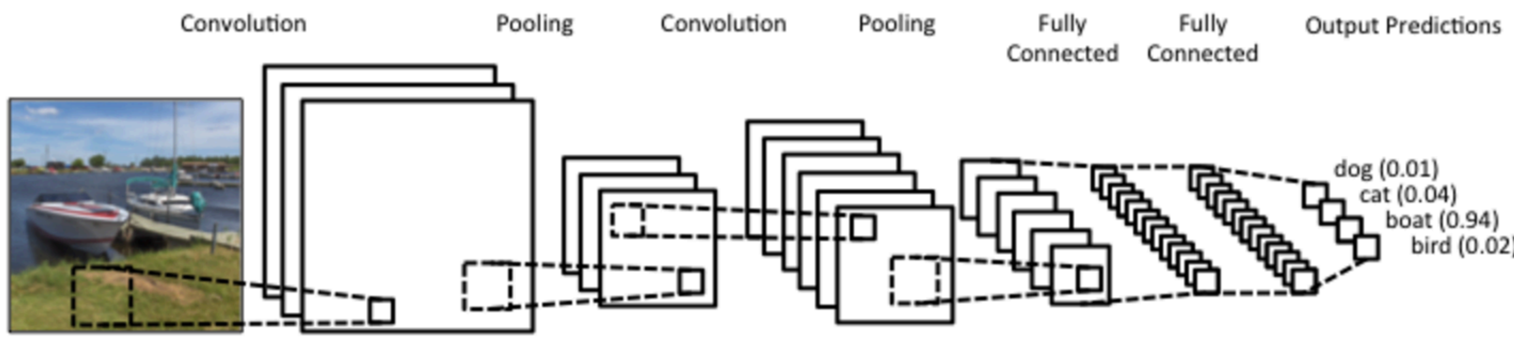


Image Filtering

- Correlation / convolution (precisely, there are subtle differences)

Correlation

$$f \otimes h = \sum_k \sum_l f(k,l)h(k,l)$$

f = Image

h = Kernel

$$f = \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix} \otimes h = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \longrightarrow$$

$$\begin{aligned} f \otimes h = {} & f_1h_1 + f_2h_2 + f_3h_3 \\ & + f_4h_4 + f_5h_5 + f_6h_6 \\ & + f_7h_7 + f_8h_8 + f_9h_9 \end{aligned}$$

Image Filtering

- Convolution – the kernel is flipped
- Flipping is skipped in convolutional network

Convolution

$$f * h = \sum_k \sum_l f(k,l)\,h(-k,-l)$$

f = Image

h = Kernel

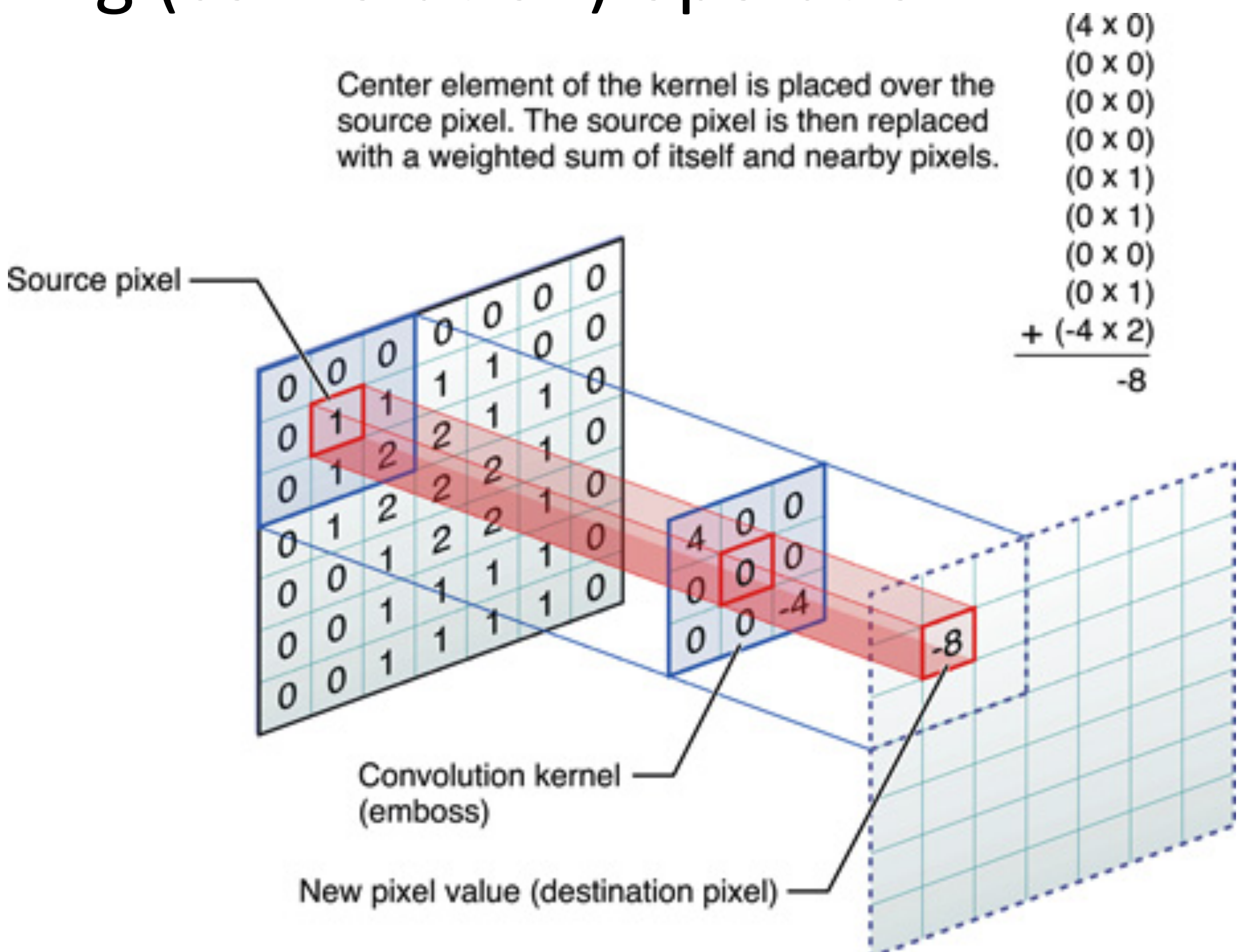
$$h = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \xrightarrow{X - flip} \begin{bmatrix} h_7 & h_8 & h_9 \\ h_4 & h_5 & h_6 \\ h_1 & h_2 & h_3 \end{bmatrix} \xrightarrow{Y - flip} \begin{bmatrix} h_9 & h_8 & h_7 \\ h_6 & h_5 & h_4 \\ h_3 & h_2 & h_1 \end{bmatrix}$$

$$f = \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix} * \begin{bmatrix} h_9 & h_8 & h_7 \\ h_6 & h_5 & h_4 \\ h_3 & h_2 & h_1 \end{bmatrix} \rightarrow$$

$$\begin{aligned} f * h = {} & f_1 h_9 + f_2 h_8 + f_3 h_7 \\ & + f_4 h_6 + f_5 h_5 + f_6 h_4 \\ & + f_7 h_3 + f_8 h_2 + f_9 h_1 \end{aligned}$$

Image Filtering

- Filtering (convolution) operation



Emboss filter: directional difference filter

Image Filtering

- Filtering (convolution) operation
- Slide the filter kernel over the entire image to produce the output (image/activation)

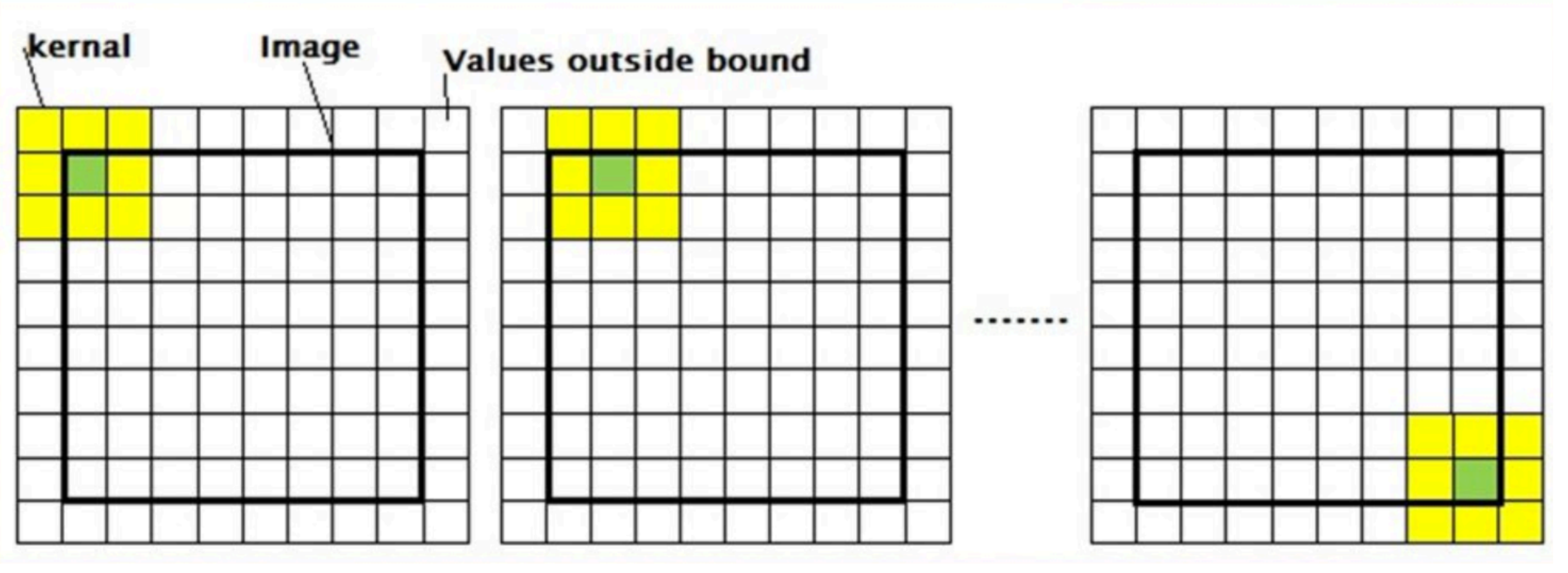


Image Filtering

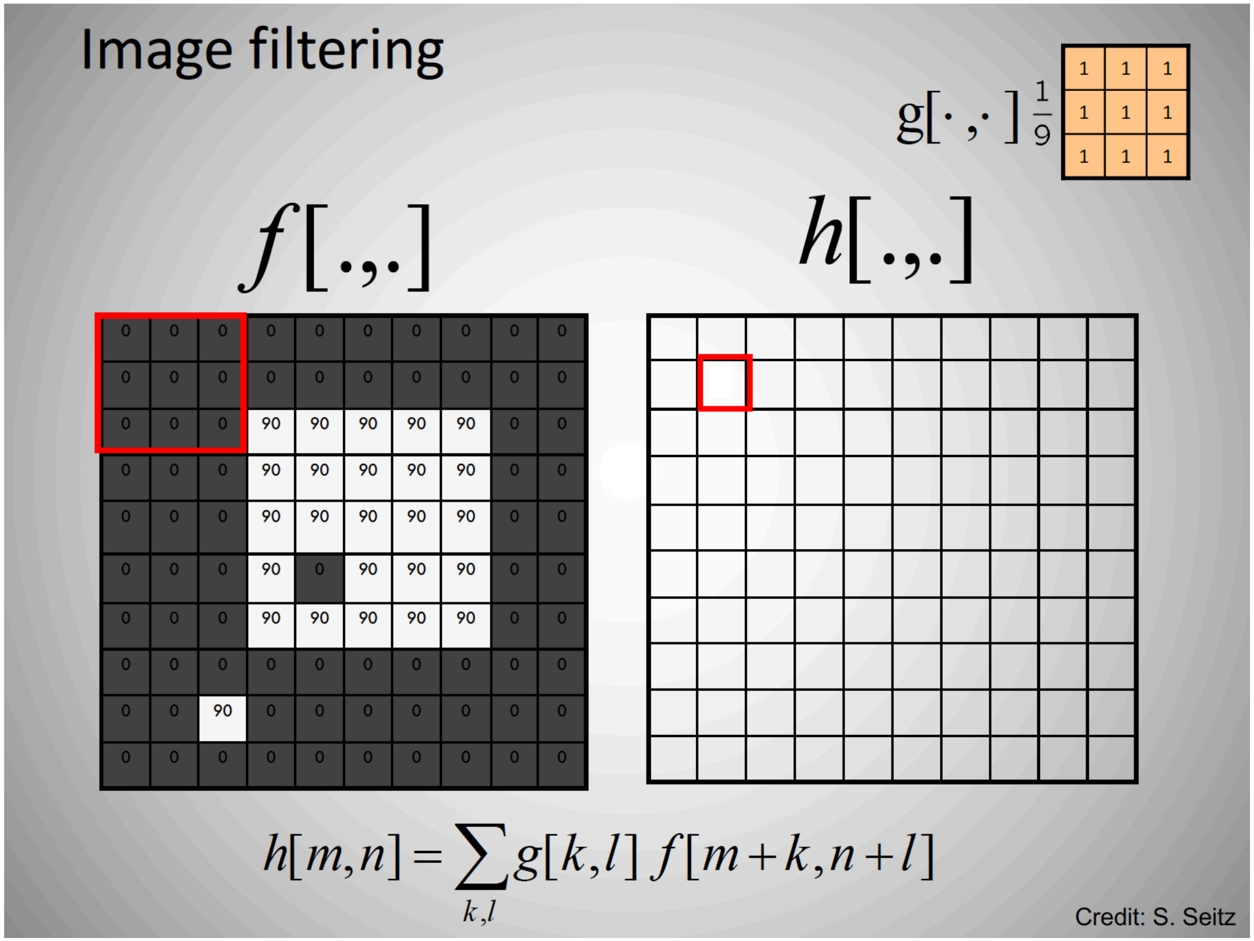


Image Filtering

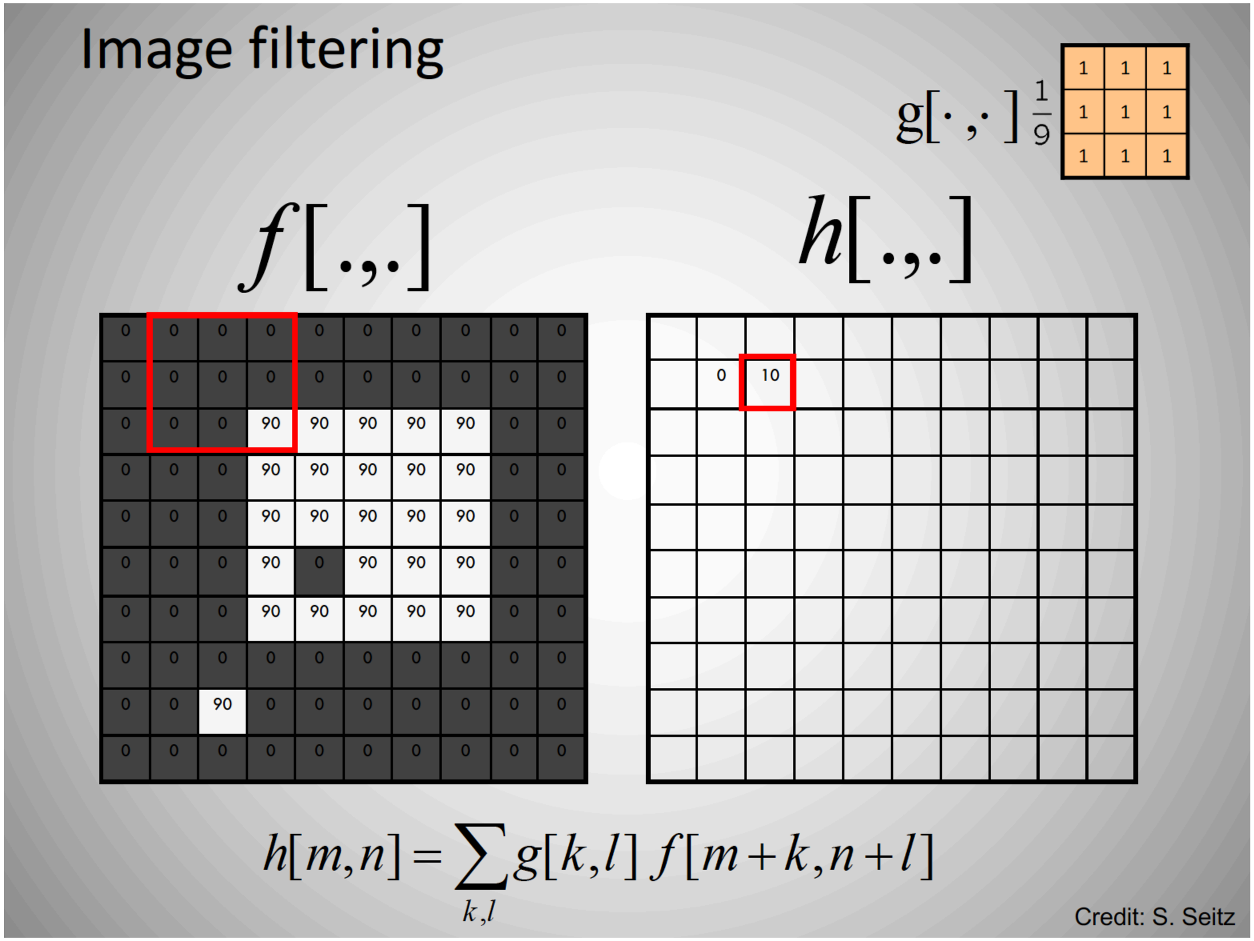


Image Filtering

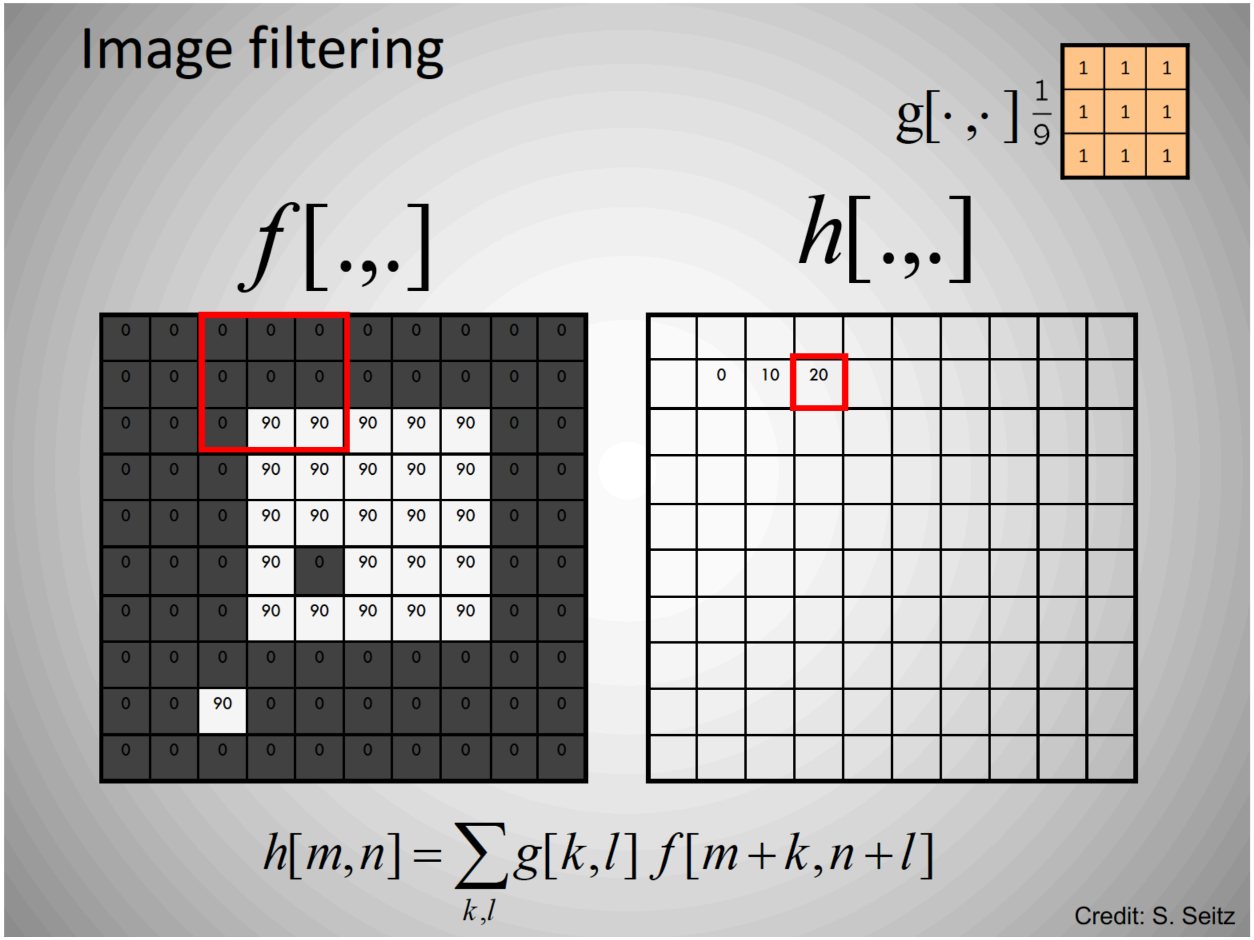


Image Filtering

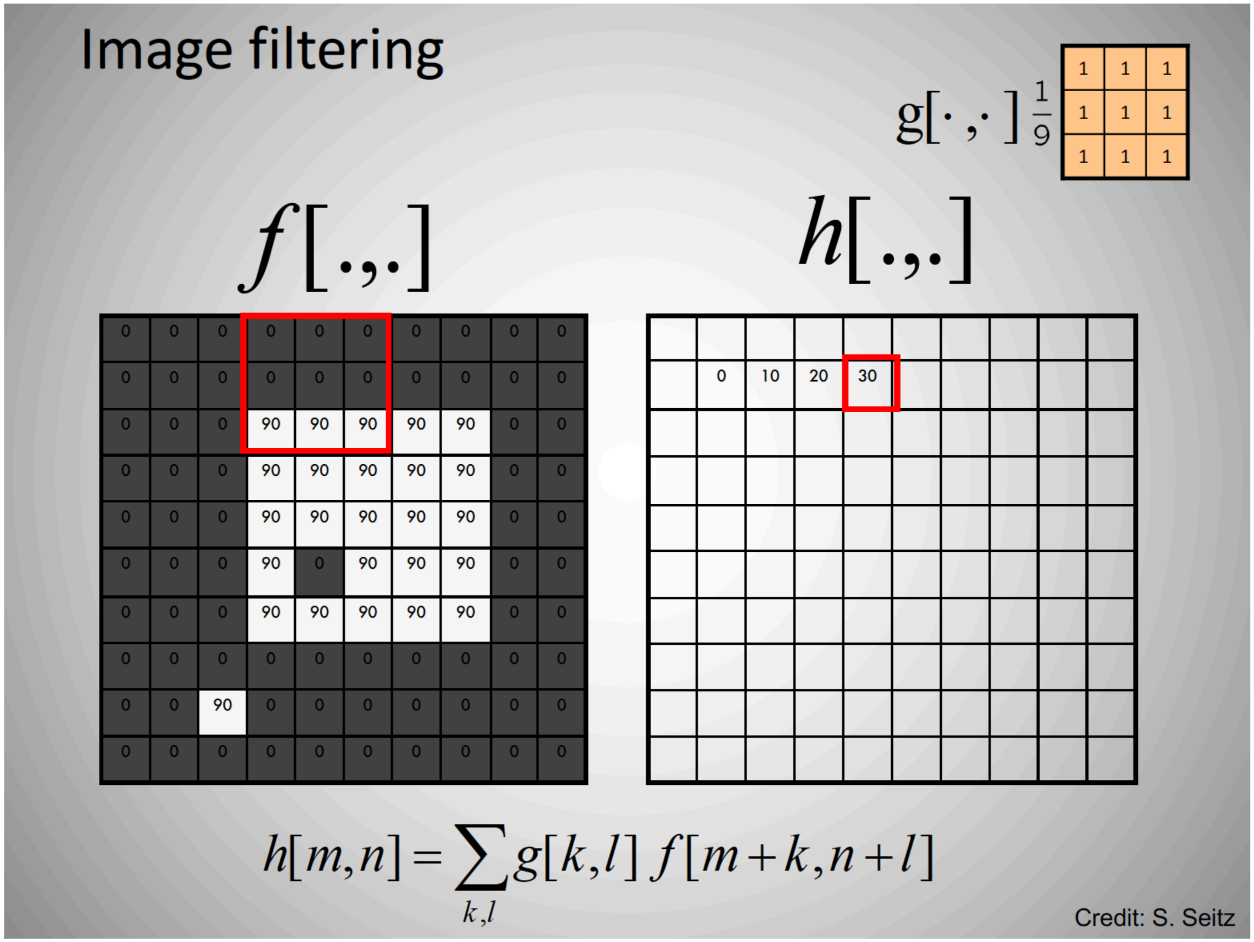
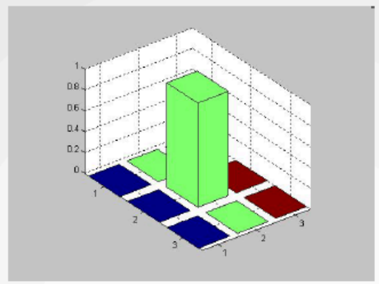


Image Filtering



$$* \begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} =$$

Image Filtering

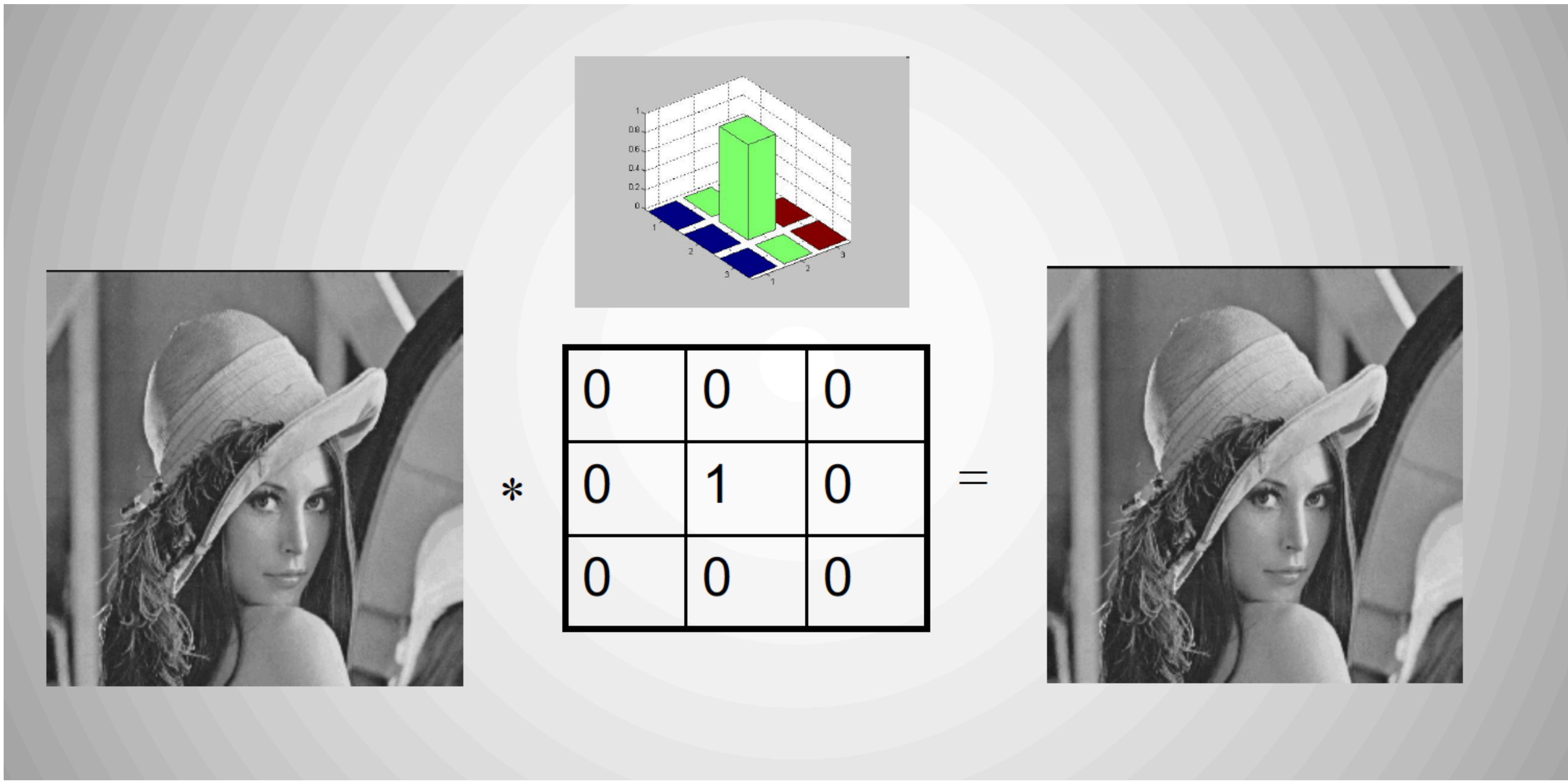
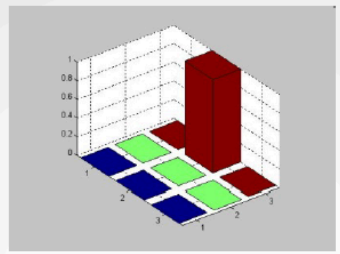


Image Filtering

$*$

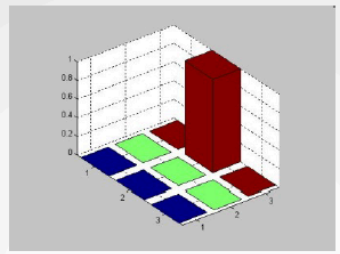


0	0	0
1	0	0
0	0	0

$=$

Image Filtering

*



0	0	0
1	0	0
0	0	0

=

Image Filtering

- Exercise: calculate the output of the following filtering (you can ignore the boundary condition)

Img =

```
[[30 40 20 30 40]
 [40 20 30 40 30]
 [20 30 40 30 40]
 [30 40 30 40 20]
 [40 30 40 20 30]]
```

kernel = $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(3 minutes)

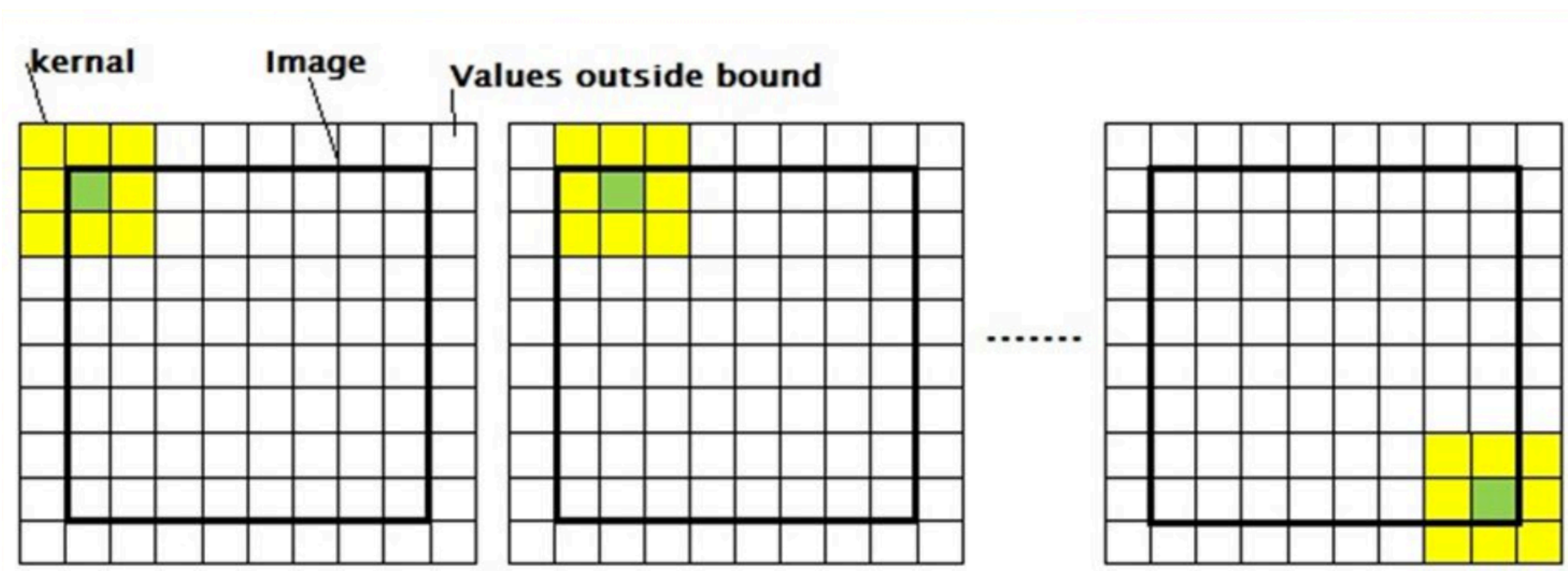
Image Filtering

- Padding to handle boundary condition
- Stride: how many pixels to shift the filter kernel in each step

Image Filtering

- Padding to handle boundary condition

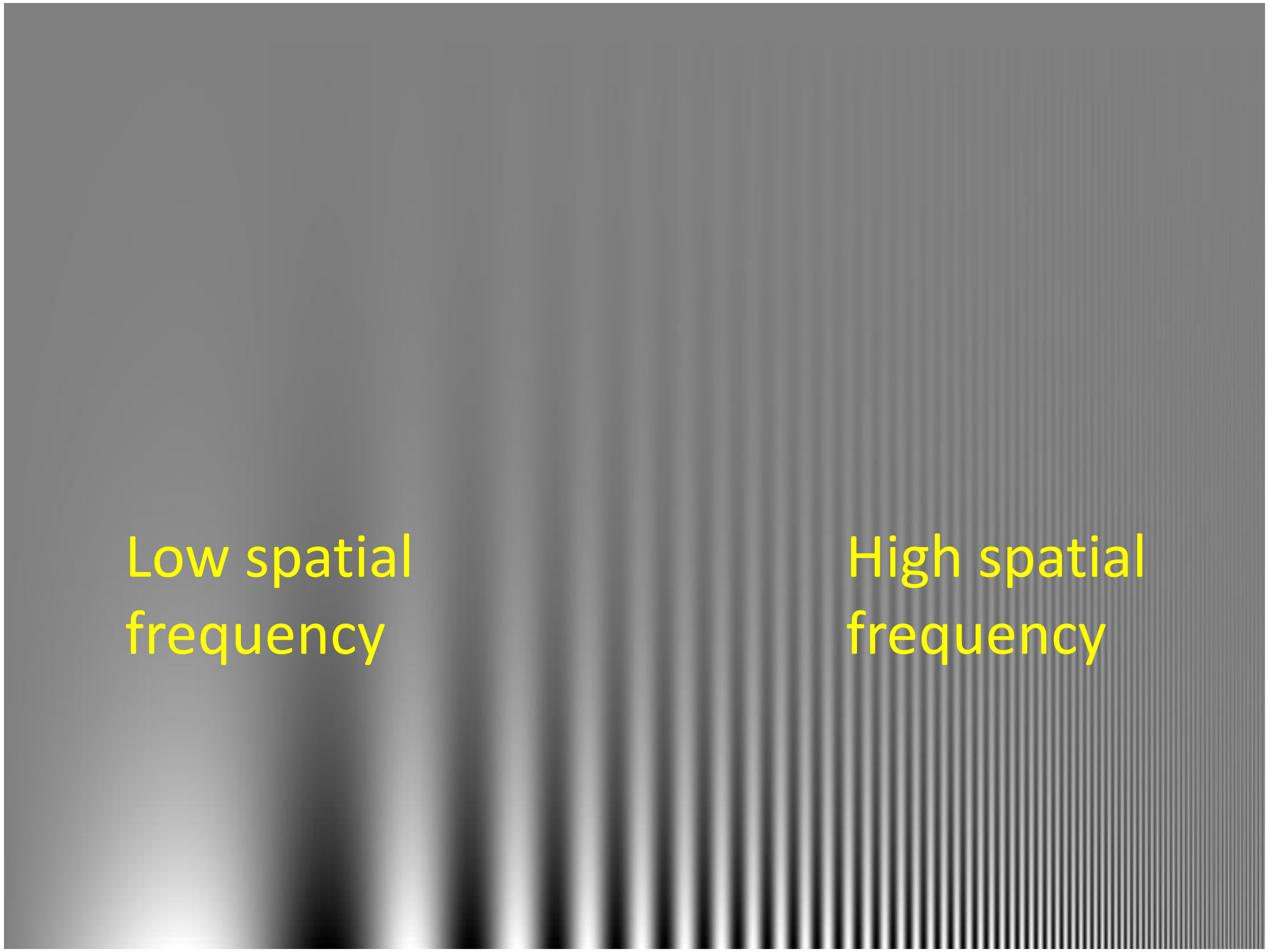
- Stride: how many pixels to shift the filter kernel in each step



Low-pass filtering

- LPF retains low spatial frequency components, remove high spatial frequency components (noise, texture)

Image Frequency



contrast

frequency

Image Frequency

- Pixel intensity at two rows

Which row has higher spatial frequency?

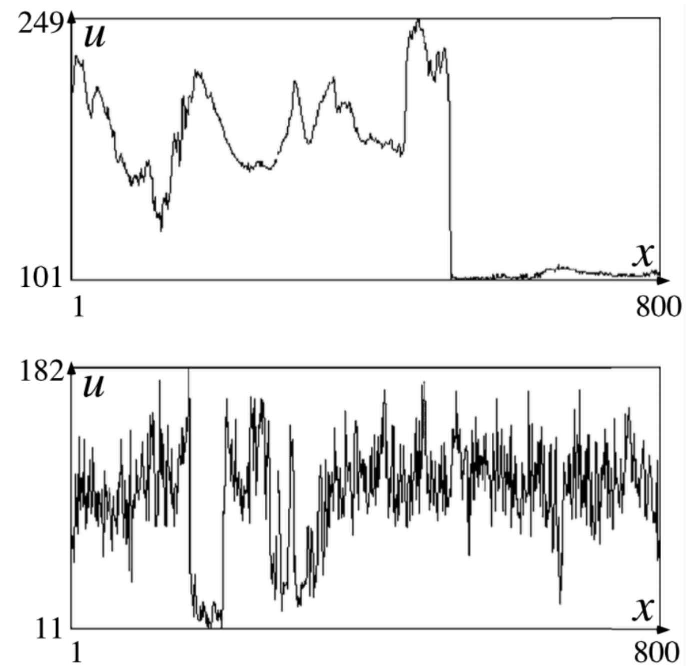
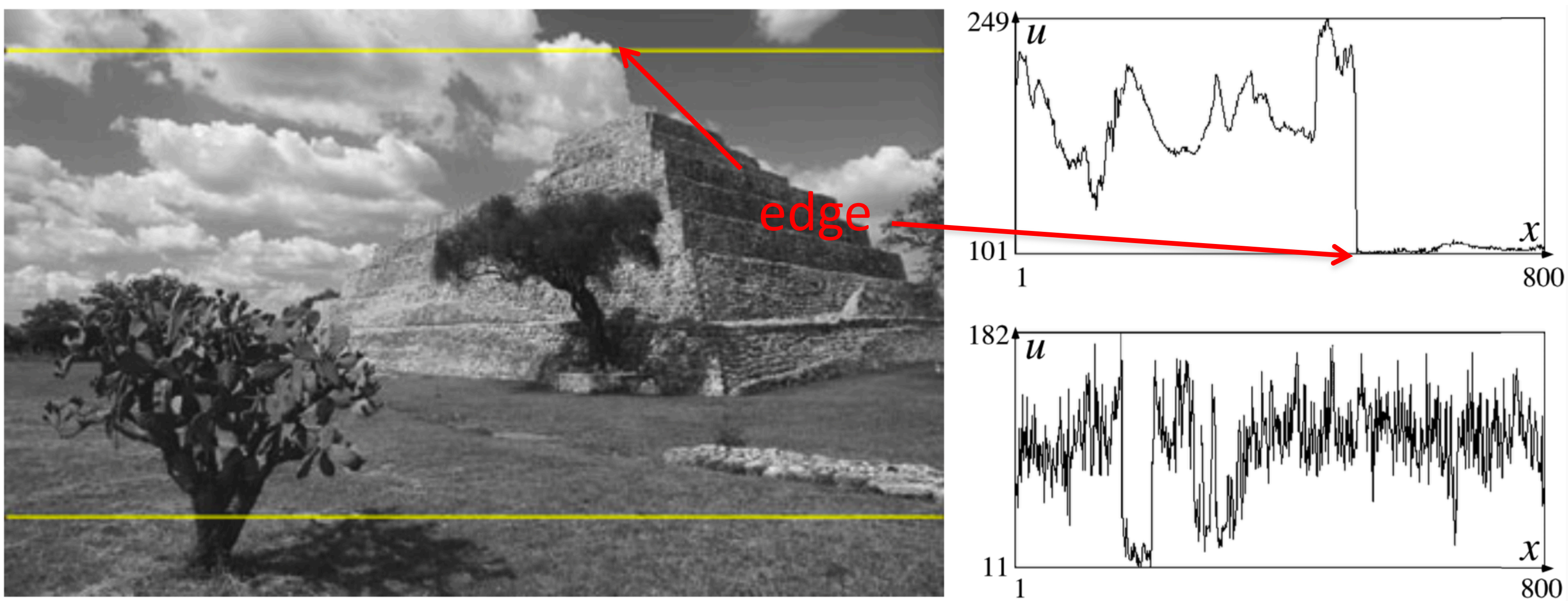


Image Frequency

- Pixel intensity at two rows

Low spatial frequency



High spatial frequency

Image Frequency

Low spatial frequency

High spatial frequency

Image Frequency

- Image frequency can be obtained quantitatively using 2D Fourier Transform, which decomposes the image into sine and cosine components

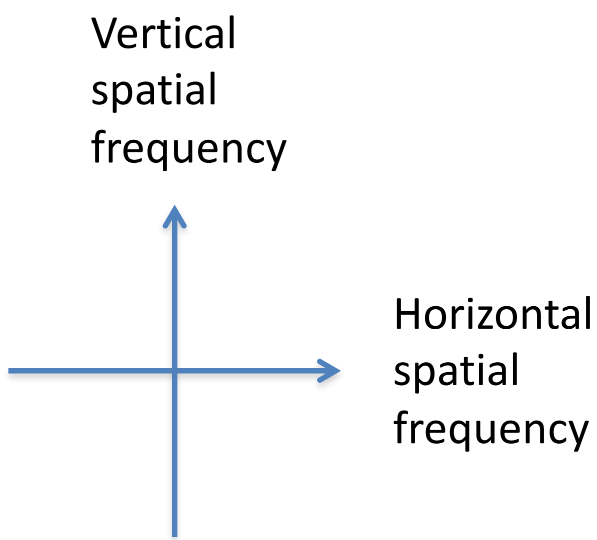
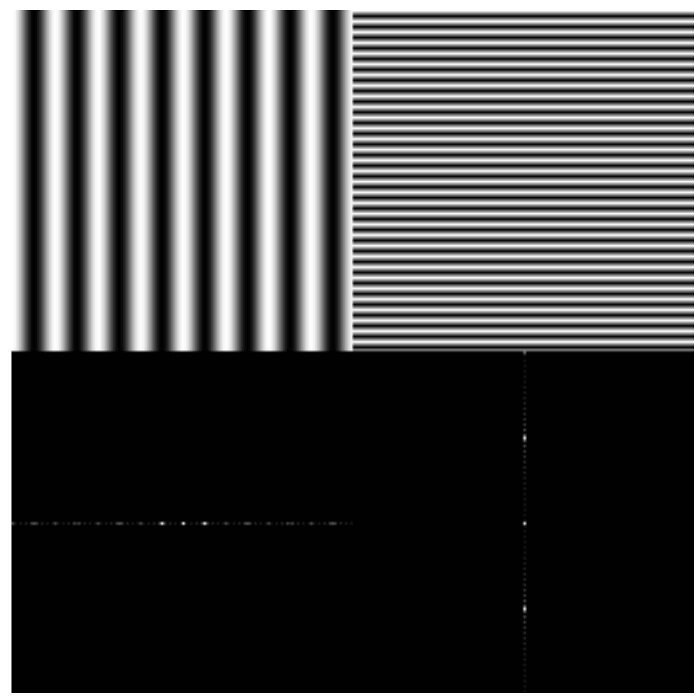


Image in frequency domain



Center point is due to the average intensity

Fourier transform (without math)

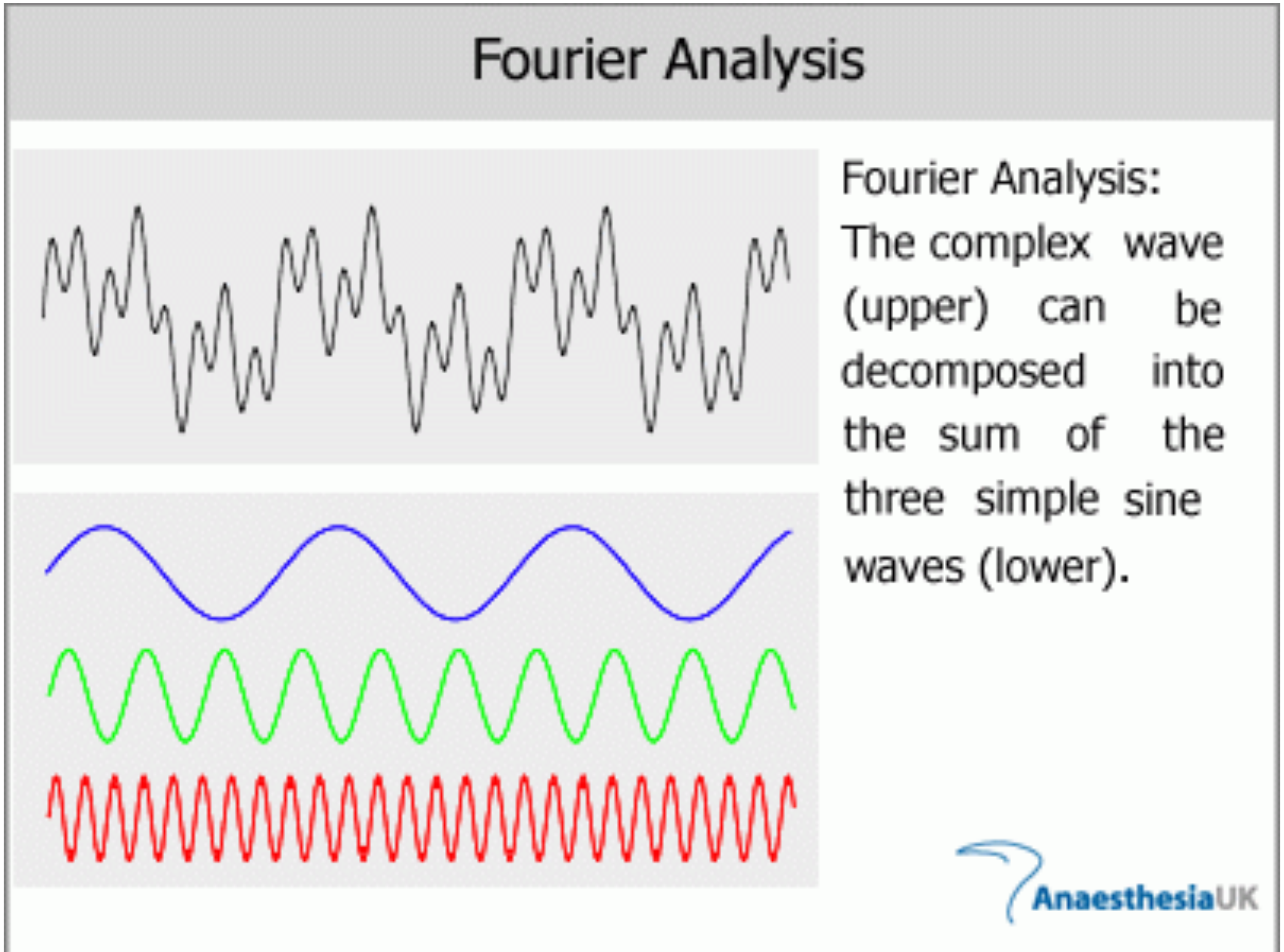


Image Frequency

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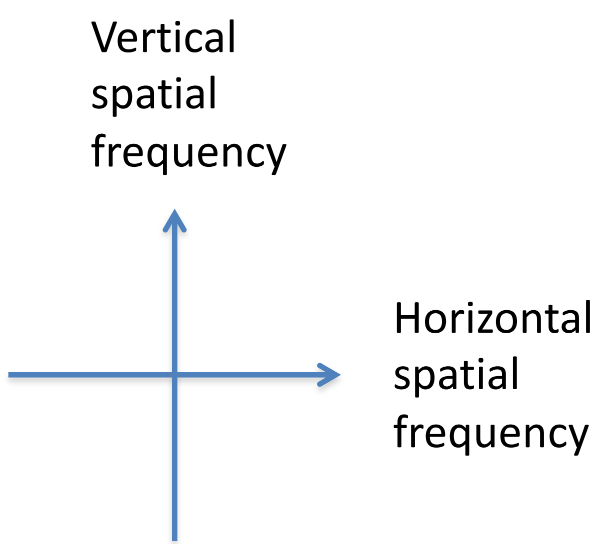

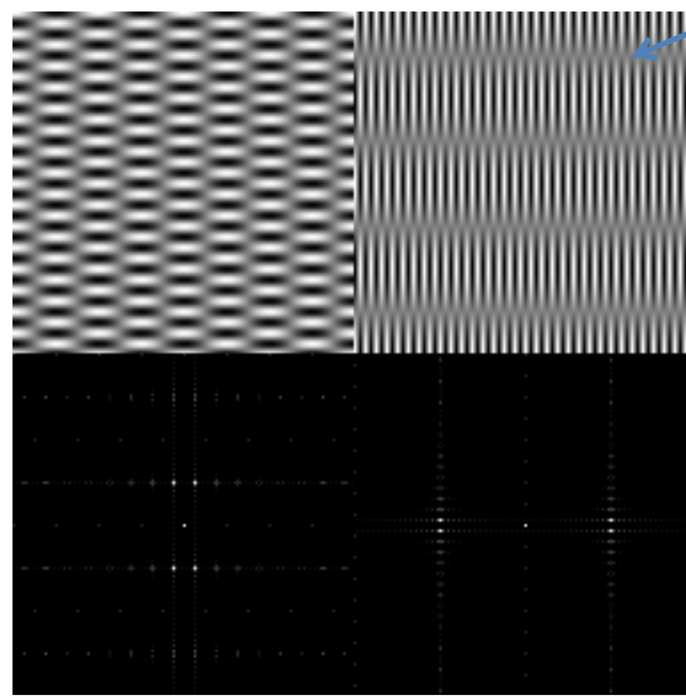

Image Frequency

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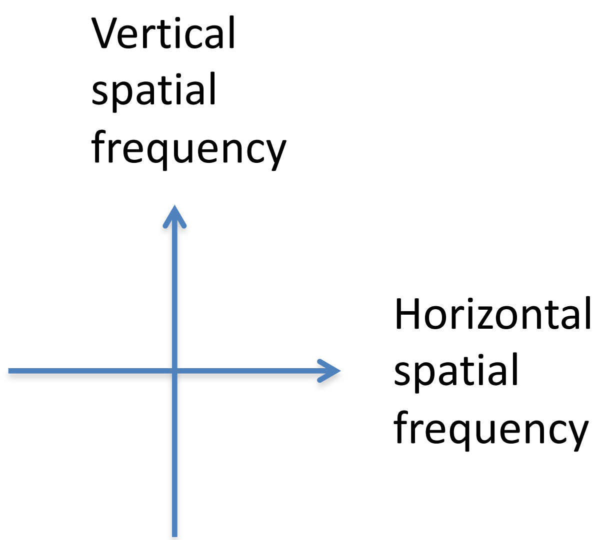
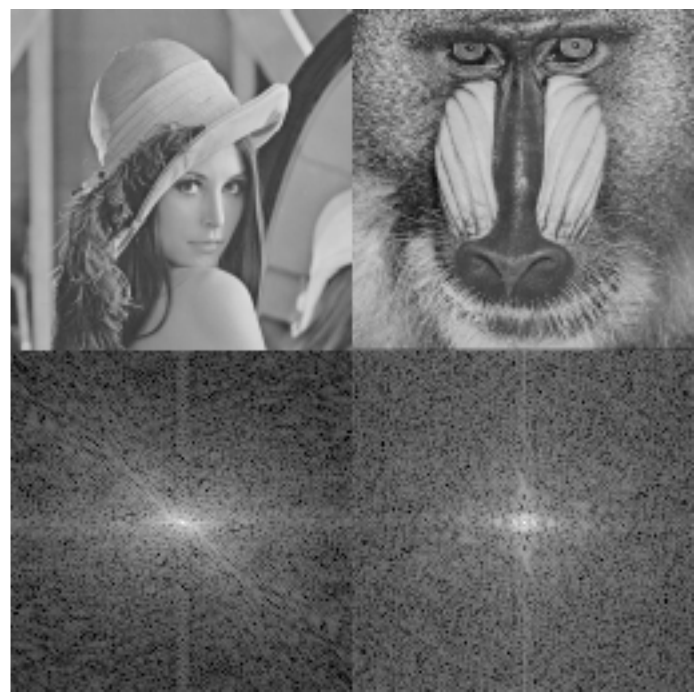




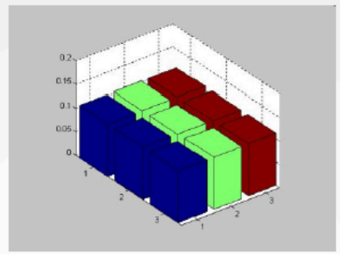
Low-pass filtering

- LPF retains low spatial frequency components, remove high spatial frequency components (noise, texture)
- An example of low-pass filter kernel

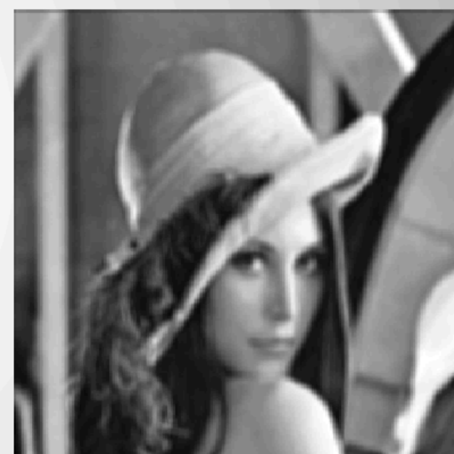
$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

(see exercise in filtering)

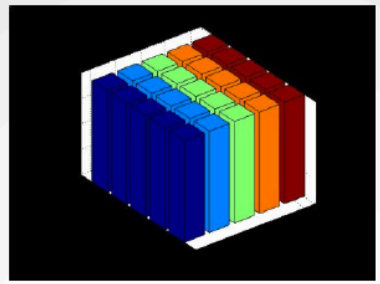
Low-pass filtering



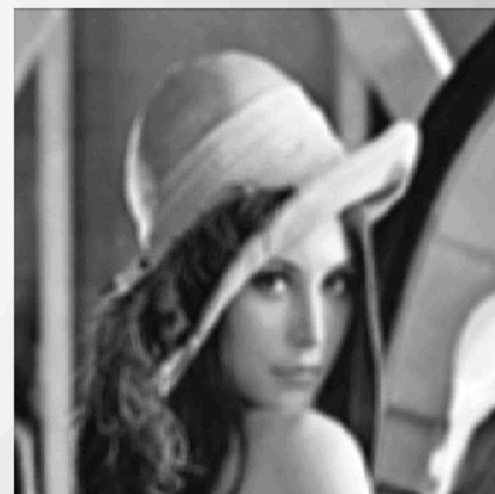
$$* \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$



Low-pass filtering

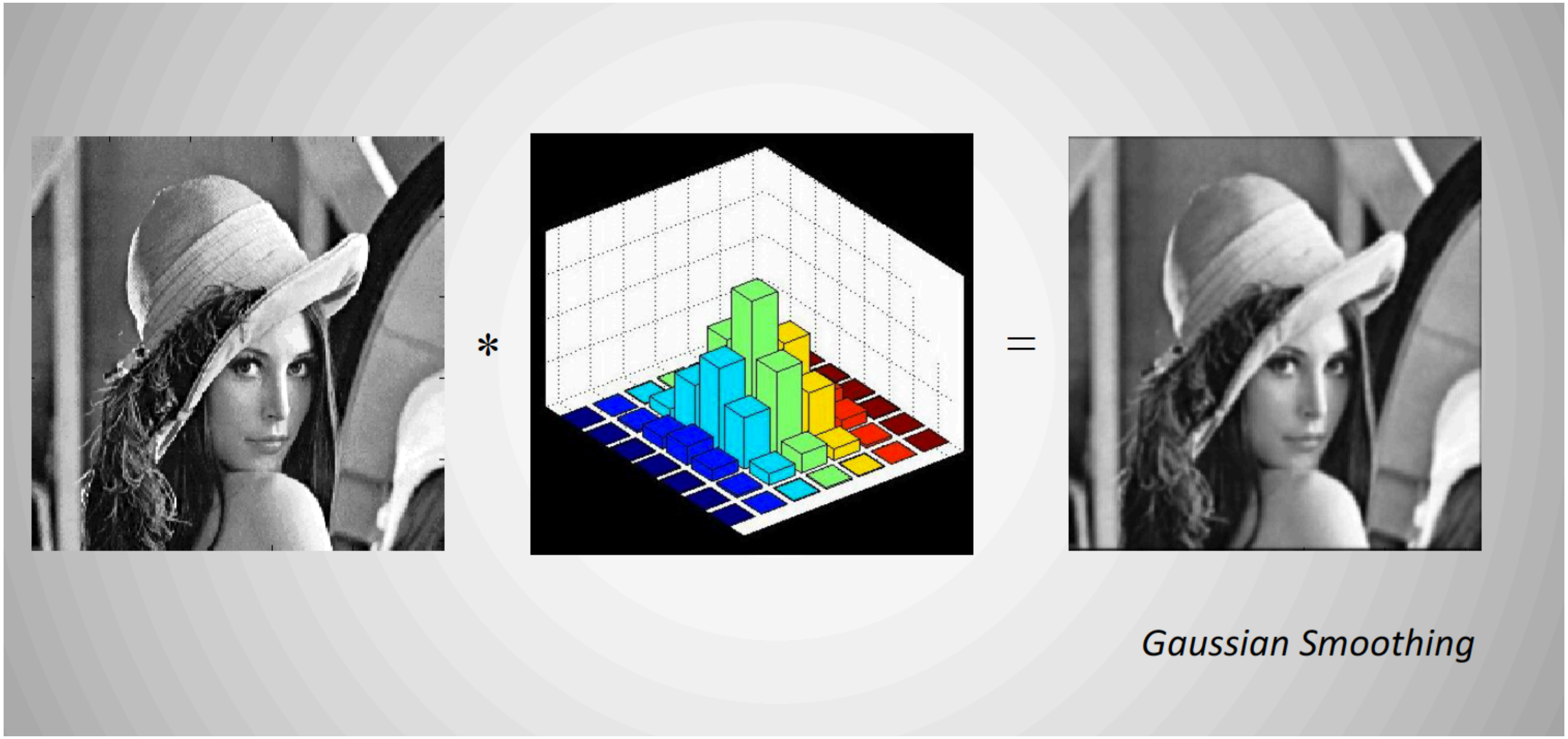


$$
* \frac{1}{25}
\begin{array}{|c|c|c|c|c|}
\hline
1 & 1 & 1 & 1 & 1 \\ \hline
1 & 1 & 1 & 1 & 1 \\ \hline
1 & 1 & 1 & 1 & 1 \\ \hline
1 & 1 & 1 & 1 & 1 \\ \hline
1 & 1 & 1 & 1 & 1 \\ \hline
\end{array}
=
$$



Larger kernel -> more blurry

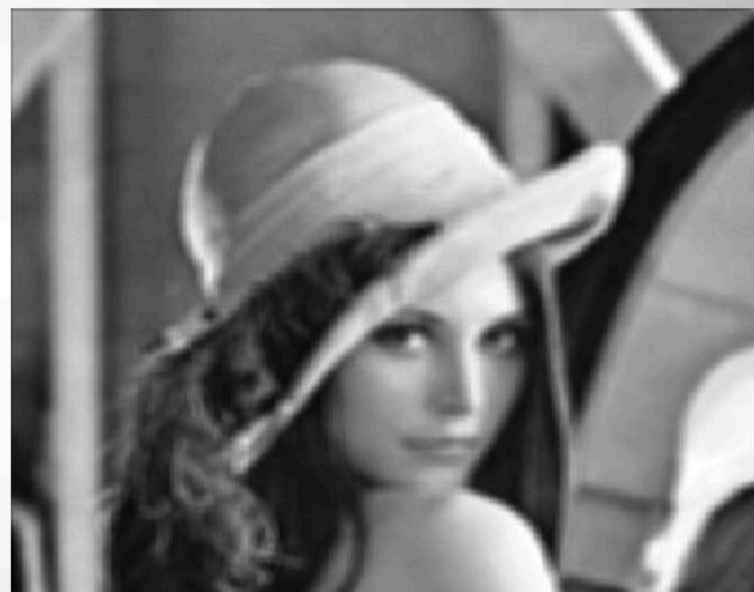
Low-pass filtering



Gaussian function: $$f(x) = ae^{-\frac{(x-b)^2}{2c^2}}$$

Low-pass filtering

Gaussian Smoothing

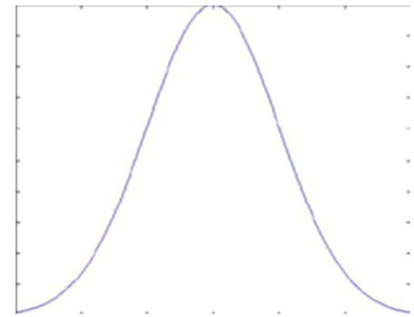


Smoothing by Averaging

Noise and denoising

Additive Gaussian Noise

$$I_{distorted} = I_{original} + n$$



n is a normally distributed

$n \sim p(n; \mu, \sigma)$

Noise: high spatial frequency

Denoising: low-pass filtering to remove the signal component with high spatial frequencies

Denoising

After additive
Gaussian Noise

After Averaging

After Gaussian Smoothing

Denoising by median filter

-**Non-linear filtering**: use the median as the filter output

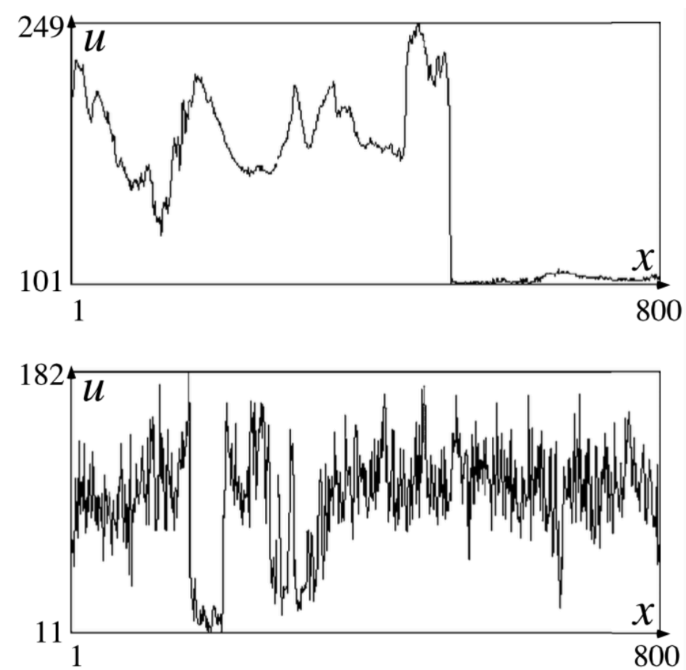
Median(2,80,6) -> 6

-More robust against outliers

-But more computationally-intensive

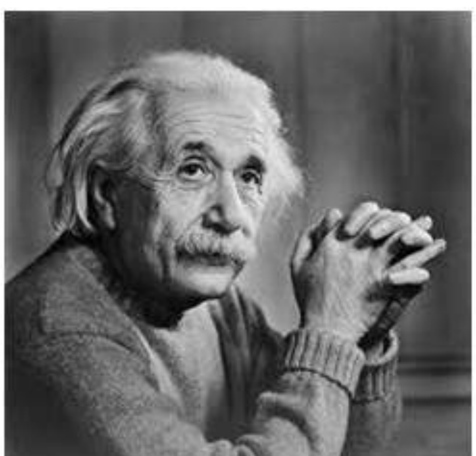
Edge detection

- Use image filtering to extract useful information: edge
- Edge: significant change in intensity values



Edge detection

- Why edge detection:
  - Edge provides important shape information: fundamental in understanding the image
  - Edge sharpen to improve visual quality



Edge detection

- Prewitt operator (as the filter kernel/mask)
  - Detect horizontal edge, vertical edge
  - Compute the difference of the neighboring pixels to indicate the likelihood of edges

Detect vertical edge	Detect horizontal edge
[[-1,0,1], [-1,0,1], [-1,0,1]]	[[-1,-1,-1], [0, 0, 0], [1, 1, 1]]

Edge detection

- Prewitt operator (as the filter kernel/mask)

[[-1,0,1],
[-1,0,1],
[-1,0,1]]

Detect vertical edge

[[-1,-1,-1],
[0, 0, 0],
[1, 1, 1]]

Detect horizontal edge

Edge detection

- Sobel operator (as the filter kernel/mask)

[[-1,0,1],
[-2,0,2],
[-1,0,1]]

Detect vertical edge

[[-1,-2,-1],
[0, 0, 0],
[1, 2, 1]]

Larger weight near the center pixel

Detect horizontal edge

Image Filtering

- Exercise: calculate the output of the following filtering (you can ignore the boundary condition)

Img =
```
[[30 40 20 30 40]
 [40 20 30 40 30]
 [20 30 40 30 40]
 [30 40 30 40 20]
 [40 30 40 20 30]]
```

kernel = $\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$

out = [[30,31,33],
[31,33,33],
[33,33,32]]

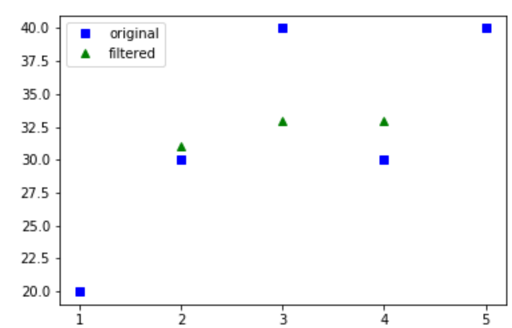
Image Filtering

- Exercise: calculate the output of the following filtering (you can ignore the boundary condition)

Img = [[30 40 20 30 40]
[40 20 30 40 30]
~~[20 30 40 30 40]~~
[30 40 30 40 20]
[40 30 40 20 30]]

kernel = $\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$

out = [[30,31,33],
~~[31,33,33],~~
[33,33,32]]



After filtering, the intensity values are smoother